ESERCIZI : Metodo del simplesso

SIMPLESSOPRIMALE(A, b, c, B)

1. $N \leftarrow \{1, \dots, m\} - B$;
2. $\overline{x} \leftarrow A_B^{-1} b_B$;
3. $\overline{y}_B \leftarrow c A_B^{-1}$;
4. $\overline{y}_N \leftarrow 0$;
5. Se $\overline{y}_B \geq 0$, allora termina con successo e restituisci $\overline{x}$ e $\overline{y}$;
6. $h \leftarrow \min\{i \in B \mid \overline{y}_i < 0\}$;
7. Sia ξ la colonna di indice h in $-(A_B^{-1})$;
8. Se $A_N \xi \leq 0$, allora termina e restituisci ξ: il problema è illimitato;
9. $k \leftarrow \arg\min\{\frac{b_i - A_i \overline{x}}{A_i \xi} \mid A_i \xi > 0 \wedge i \in N\}$;
10. $B \leftarrow B \cup \{k\} - \{h\}$;
11. Torna al punto 1.

Esercizio 3.1. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\min 3x_1 - x_2$$

$$x_1 + 1 \geq 1 \qquad x_2 + 1 \geq 1$$
$$x_2 \leq 2x_1 + 2 \qquad 2x_2 + 2 \geq x_1$$
$$x_2 + 2 \geq x_1$$

Si parta dalla base ammissibile corrispondente ai vincoli della prima riga.

$x_1 = x$
$x_2 = y$

$\max -3x + y \qquad c\,[-3, 1]$

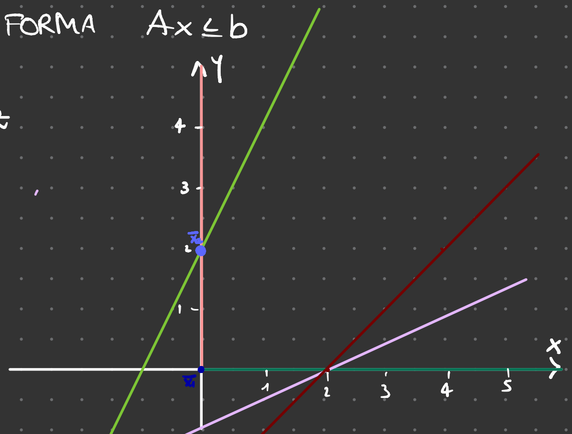
TRASFORMO I VINCOLI NELLA FORMA $Ax \leq b$

1 $-x \leq 0$ 2 $-y \leq 0$

3 $-2x + y \leq 2$ ~~4 $x - 2y \leq 2$~~

5 $x - y \leq 2$

$$A = \begin{array}{c|cc} & x & y \\ 1 & & \\ 2 & & \\ 3 & & \\ 4 & & \end{array} \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -2 & 1 \\ 1 & -1 \end{bmatrix} \qquad b = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$



$B_1 = \{1, 2\}$

$$A_{B_1} = \begin{array}{c} 1 \\ 2 \end{array}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = A_{B_1}^{-1} \qquad \bar{x}_1 = A_{B_1}^{-1} b_{B_1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bar{y}_{B_1} = c A_{B_1}^{-1} = [-3 \;\; 1] \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = [3 \;\; -1] \qquad \bar{y} = [3 \;\; \underset{\downarrow}{-1} \;\; 0 \;\; 0]$$

$h = 2$

$$-A_{B_1}^{-1} = \begin{bmatrix} 1 & \boxed{0} \\ 0 & \boxed{1} \end{bmatrix} \rightarrow \xi \qquad A_N \xi = \begin{array}{c} 3 \\ 4 \end{array}\begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$k = 3$

$B_2 = \{1, 3\}$

$$A_{B_2} = \begin{array}{c} 1 \\ 3 \end{array}\begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix} \qquad A_{B_2}^{-1} = \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix} \qquad \bar{x}_2 = A_{B_2}^{-1} b_{B_2} = \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

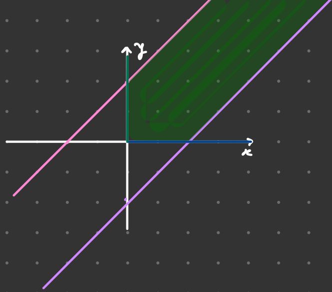
$$\bar{y}_{B_2} = c A_{B_2}^{-1} = [-3 \;\; 1] \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix} = [1 \;\; 1]$$

$\rightarrow \bar{y}_{B_2}$ non è negativo

x è soluzione ottima per il primale
y è soluzione ottima per il duale

valore ottimo $cx = 2$

max $x+2y$



parto dai primi due vincoli

1) $x \geq 0$ $\quad -x \leq 0$

2) $y \geq 0$ $\quad -y \leq 0$

3) $x-y+2 \geq 0$ $\quad -x+y \leq 2$

4) $-x+y+2 \geq 0$ $\quad x-y \leq 2$

$$A \begin{vmatrix} -1 & 0 \\ 0 & -1 \\ -1 & 1 \\ 1 & -1 \end{vmatrix} \quad b = \begin{vmatrix} 0 \\ 0 \\ 2 \\ 2 \end{vmatrix}$$

I iterazione

$B_0 \{1,2\}$

$$A = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \quad A_0^{-1} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \quad x_0 = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix} \quad y_0 = \begin{vmatrix} 1 & 2 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} \underset{h=1}{\underset{\downarrow}{-1}} & -2 \end{vmatrix}$$

$$-A_0^{-1} = \begin{vmatrix} \boxed{\begin{matrix} 1 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ 1 \end{matrix} \end{vmatrix} \to \xi \qquad A_N \xi = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} -1 \\ 1 \end{vmatrix} \quad k=4$$

II iterazione

$B_1 = \{2,4\}$

$$A_1 = \begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix} \quad A_1^{-1} = \frac{1}{0+1} \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} \quad x_1 = \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 2 \end{vmatrix} = \begin{vmatrix} 2 \\ 0 \end{vmatrix} \quad y_1 = \begin{vmatrix} 1 & 2 \end{vmatrix} \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} \underset{h=2}{\underset{\downarrow}{-3}} & 1 \end{vmatrix}$$

$$-A_1^{-1} = \begin{vmatrix} \boxed{\begin{matrix} 1 \\ 1 \end{matrix}} & \begin{matrix} -1 \\ 0 \end{matrix} \end{vmatrix} \to \xi \qquad A_N \xi = \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \end{vmatrix}$$

termino restituisco $\xi = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$
il problema illimitato
$A_N \xi \leq 0$

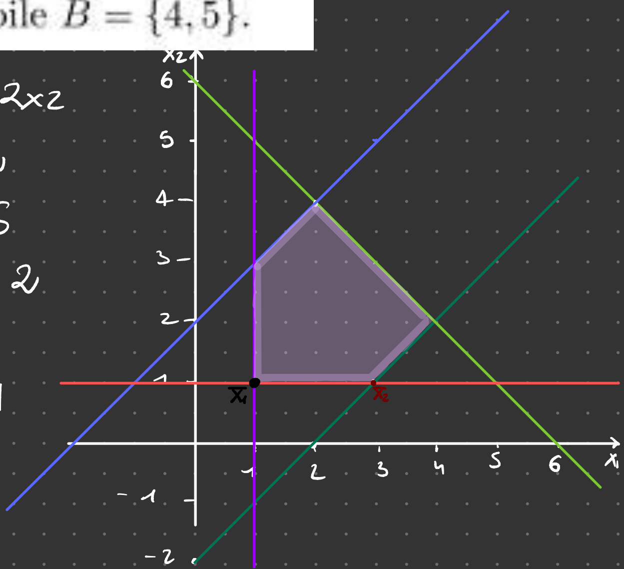
3.2.1 Temi d'esame 2013

Esercizio 3.3. Si risolva tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$
\begin{aligned}
\max \quad & x_1 - 2x_2 \\
& x_2 \geq x_1 - 2 \\
& x_2 \leq 6 - x_1 \\
& x_2 \leq x_1 + 2 \\
& x_1 \geq 1 \\
& x_2 \geq 1
\end{aligned}
$$

Si parta dalla base ammissibile $B = \{4, 5\}$.

$$
\begin{aligned}
& \max \quad x_1 - 2x_2 \\
1 \quad & x_1 - x_2 \leq 2 \\
2 \quad & x_1 + x_2 \leq 6 \\
3 \quad & -x_1 + x_2 \leq 2 \\
4 \quad & -x_1 \leq -1 \\
5 \quad & -x_2 \leq -1
\end{aligned}
$$



$$
A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ -1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 6 \\ 2 \\ -1 \\ -1 \end{bmatrix}
$$

$B_1 = \{4, 5\}$

$$
A_{B_1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad A_{B_1}^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \bar{x}_1 = A_{B_1}^{-1} b_{B_1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
$$

$$
\bar{y}_1 = c A_{B_1}^{-1} = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \end{bmatrix} \qquad y = \begin{bmatrix} 0 & 0 & 0 & \underset{\downarrow}{-1} & 2 \end{bmatrix}
$$

$h = 4$

$$-A^{-1}_{B_1} = \begin{bmatrix} \boxed{\begin{matrix}1\\0\end{matrix}} & \begin{matrix}0\\1\end{matrix} \end{bmatrix}$$

$$\xi$$

$$A_N \xi = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\frac{b_i - A_i \bar{x}}{A_i \xi} \quad \xrightarrow{i=1} \quad \frac{2 - \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{1} = 2 \qquad K = 1$$

$$\xrightarrow{i=2} \quad \frac{6 - \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{1} = 4$$

$B_2 = \{1, 5\}$

$$A_{B_2} = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} \qquad A^{-1}_{B_2} = \begin{bmatrix} +1 & -1 \\ 0 & -1 \end{bmatrix} \qquad \bar{x} = A^{-1}_{B_2} \bar{b}_{B_2} = \begin{bmatrix} +1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\bar{y}_{B_2} = c A^{-1}_{B_2} = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Soluzione ottime

$$\bar{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \bar{y} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Esercizio 3.4. Si risolva tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\begin{aligned} \min \quad & x_1 + x_2 \\ & x_1 \geq -1 \\ & x_2 \geq x_1 - 1 \\ & x_2 \leq x_1 + 1 \\ & x_1 \leq 3 \\ & x_2 \leq 4 \\ & x_2 \geq -1 \end{aligned}$$

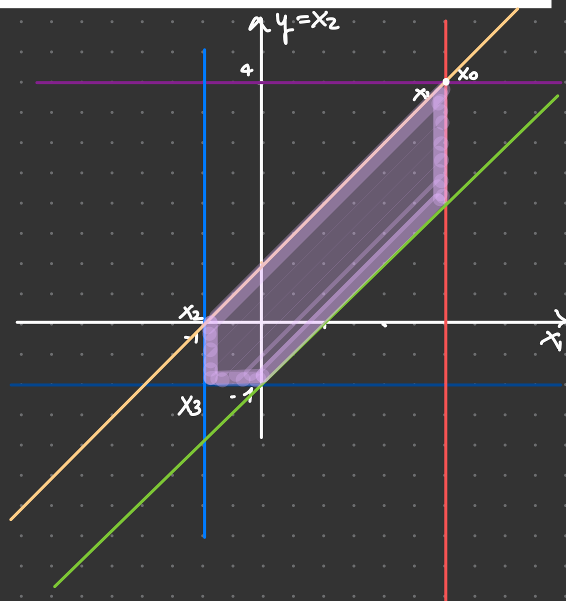
Si parta dalla base ammissibile $B = \{4, 5\}$.

$\max \; -x_1 - x_2 \qquad c\,[-1 \;\; -1]$

1 $\; -x_1 \le 1$ $\qquad$ 4 $\; x_1 \le 3$

2 $\; x_1 - x_2 \le 1$ $\qquad$ 5 $\; x_2 \le 4$

3 $\; -x_1 + x_2 \le 1$ $\qquad$ 6 $\; -x_2 \le 1$



$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$B_0 = \{4, 5\}$

$$A_{B_0}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{\underset{\substack{\| \\ a \cdot d - b \cdot c}}{\det}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\overline{x_0} = A_{B_0}^{-1} b_{B_0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\overline{y_0} = c A_{B_0}^{-1} = [-1 \;\; -1] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [-1 \;\; -1] \qquad y = [0 \; 0 \; 0 \; -1 \; -1 \; 0] \rightarrow h = 4$$

$$-A_{B_0}^{-1} = \begin{bmatrix} \boxed{\begin{matrix} -1 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ -1 \end{matrix} \end{bmatrix} \rightarrow \xi$$

$$A_{N_0} \xi = \begin{array}{c} 1 \\ 2 \\ 3 \\ 6 \end{array} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\frac{b_i - A_i x}{A_i \xi} \begin{cases} \xrightarrow{i=1} \dfrac{1 - \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{1} = 1 + 3 = 4 \\ \xrightarrow{i=3} \dfrac{1 - \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{1} = \dfrac{1+3-4}{1} = 0 \end{cases} \quad K=3$$

$B_1 = \{3, 5\}$

$$A_{B_1} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \qquad A_{B_1}^{-1} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\overline{x_1} = A_{B_1}^{-1} b_{B_1} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\overline{y_1} = c A_{B_1}^{-1} = \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \end{bmatrix} \qquad h=5$$

$$-A_{B_1}^{-1} = \begin{bmatrix} 1 & \boxed{-1} \\ 0 & \boxed{-1} \end{bmatrix} \rightarrow \xi$$

$$A_{N_1} \xi = \begin{array}{c} 1 \\ 2 \\ 4 \\ 6 \end{array} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\frac{b_i - A_i x}{A_i \xi} \begin{cases} \xrightarrow{1} \dfrac{1 - \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{1} = 1+3 = 4 \quad K=1 \\ \xrightarrow{6} \dfrac{1 - \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}}{1} = 1+4 = 5 \end{cases}$$

$B_2 = \{1, 3\}$

$$A_{B_2} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \qquad A_{B_2}^{-1} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\overline{x_2} = A_{B_2}^{-1} b_{B_2} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\overline{y_2} = c A_{B_2}^{-1} = \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \end{bmatrix} \qquad h=3$$

$$-A_{B_2}^{-1} = \begin{bmatrix} 1 & \boxed{0} \\ 1 & \boxed{-1} \end{bmatrix} \rightarrow \xi$$

$$A_{N_2} \xi = \begin{array}{c} 2 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\frac{b_i - A_i x}{A_i \xi} \quad \xrightarrow{2} \quad \frac{1 - [1\ -1]\begin{bmatrix}-1\\-1\end{bmatrix}}{} = 1$$

$$\xrightarrow{6} \quad \frac{1 - [0\ -1]\begin{bmatrix}-1\\-1\end{bmatrix}}{1} = 0 \qquad k = 6$$

$B_3 = \{1, 6\}$

$A_{B_3} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \qquad A_{B_3}^{-1} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

$\overline{x_3} = A_{B_3}^{-1} b_{B_3} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

$\overline{y_3} = c A_{B_3}^{-1} = [-1\ \ -1]\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} = [1\ \ 1]$

$\overline{x}_3$ e $\overline{y}_3$ soluzioni ottime

Esercizio 3.5. Si risolva tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\begin{aligned} \min\ & 2x_1 + x_2 \\ & x_1 \geq -1 \\ & x_2 \geq -1 \\ & x_2 \leq 2 \\ & 2x_2 \leq 2 - x_1 \\ & x_2 + 3 \geq 2x_1 \end{aligned}$$

Si parta dalla base ammissibile $B = \{4, 5\}$.

$\max\ -2x - y$

1) $-x \leq 1$

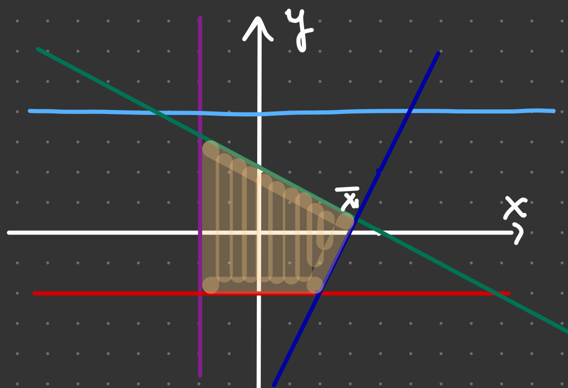
2) $-y \leq 1$

3) $y \leq 2$

4) $x + 2y \leq 2$

5) $2x - y \leq 3$

y



$$A \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix} \quad b \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$

$$B_1 = \{4, 5\}$$

$$A_1 = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \quad A_1^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

$$\overline{x}_1 = A_1^{-1} b_1 = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 17/5 \\ 1/5 \end{bmatrix}$$

$$\overline{y}_1 = c A_1^{-1} = \begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix} = \begin{bmatrix} -4/5 & \cdot 1/5 \end{bmatrix}$$

$h = 4$

$$-A_1^{-1} = \begin{bmatrix} \boxed{\begin{matrix} -1/5 \\ -2/5 \end{matrix}} & \begin{matrix} -2/5 \\ +1/5 \end{matrix} \end{bmatrix}$$

ζ

$$A_N \zeta_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1/5 \\ -2/5 \end{bmatrix} = \begin{bmatrix} 1/5 \\ 2/5 \\ -2/5 \end{bmatrix}$$

$$\frac{b_i - A_i x}{A_i \zeta_1}$$

$$\xrightarrow{i=1} \frac{1 - \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 17/5 \\ 1/5 \end{bmatrix}}{1/5} = \frac{1 + \frac{17}{5}}{1/5} = \frac{22}{\cancel{5}} \cdot \frac{\cancel{5}}{1} = 22$$

$$\xrightarrow{i=2} \frac{1 - \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} 17/5 \\ 1/5 \end{bmatrix}}{2/5} \quad \frac{1 + 1/5}{2/5} \quad \frac{\cancel{6}^{3}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{2}} = 3$$

$k = 2$

$$B_2 = \{2, 5\}$$

$$A_2 = \begin{bmatrix} 0 & -1 \\ 2 & -1 \end{bmatrix} \quad A_2^{-1} = \begin{bmatrix} -1/2 & 1/2 \\ -1 & 0 \end{bmatrix} \quad \overline{x}_2 = A_2^{-1} b_2 = \begin{bmatrix} -1/2 & 1/2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\overline{y}_2 = c A_2^{-1} = \begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} -1/2 & 1/2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$h = 5$

$$-A_2^{-1} = \begin{bmatrix} 1/2 & \boxed{-1/2} \\ 1 & 0 \end{bmatrix} \qquad \xi$$

$$A_N \xi = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix} \qquad k=1$$

$B_3 = \{1, 2\}$

$$A_3^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \bar{x}_3 = A_3^{-1} b_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\bar{y}_3 = c\, A_3^{-1} = \begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

$\bar{x}_3$ e $\bar{y}_3$ sono soluzioni ottime

Esercizio 3.6. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\begin{aligned} \max \quad & x_1 + x_2 \\ & x_2 \geq -1 \\ & x_1 \geq 0 \\ & x_2 \leq x_1 \\ & x_1 - 1 \leq x_2 \end{aligned}$$

Si parta dalla base ammissibile $B = \{1, 2\}$.

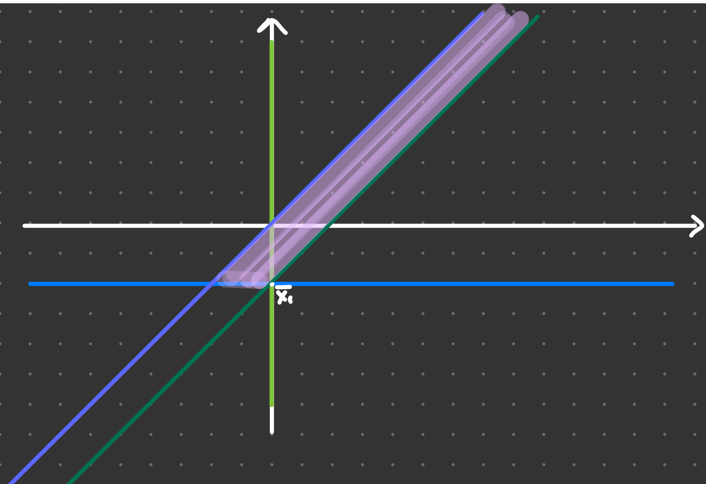
$[1 \;\; 1]$

1) $-y \leq 1$

2) $-x \leq 0$

3) $-x + y \leq 0$

4) $x - y \leq 1$



$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$B_1 = \{1, 2\}$

$$A_1^{-1} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad \bar{x}_1 = A_1^{-1} b_1 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\bar{y}_1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} \underset{h=1}{-1} & -1 \end{bmatrix}$$

$$-A_1^{-1} = \begin{bmatrix} \boxed{\begin{matrix} 0 \\ 1 \end{matrix}} & \begin{matrix} 1 \\ 0 \end{matrix} \end{bmatrix} \qquad A_N \xi = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad K = 3$$

ξ

$B_2 = \{2, 3\}$

$$A_2^{-1} = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \qquad \bar{x}_2 = A_2^{-1} b_2 = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bar{y}_2 = c A_2^{-1} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \underset{h=2}{-2} & 1 \end{bmatrix}$$

$$-A_2^{-1} = \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 1 \end{matrix}} & \begin{matrix} 0 \\ -1 \end{matrix} \end{bmatrix} \qquad A_N \xi = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

PROBLEMA È ILLIMITATO

ξ

Esercizio 3.8. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\max \quad x_2 - x_1$$

$$x_1 \le 1 \qquad\qquad x_1 + x_2 \le 2$$
$$2x_2 + 1 \ge x_1 \qquad\qquad 2x_2 \le 4x_1 + 2$$
$$-x_2 \ge -1$$

Si parta dalla base ammissibile costituita dai vincoli $x_1 \le 1$ e $2x_2 + 1 \ge x_1$.

$\max \quad y - x \qquad [-1 \quad 1]$

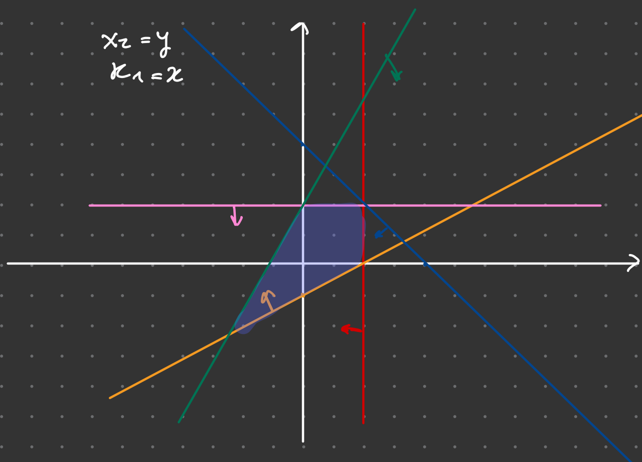
1) $x \le 1$

2) $x - 2y \le 1$

3) $y \le 1$

4) $x + y \le 2$

5) $-4x + 2y \le 2$



$A \begin{bmatrix} 1 & 0 \\ 1 & -2 \\ 0 & 1 \\ 1 & 1 \\ -4 & 2 \end{bmatrix} \qquad b \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}$

$B_0 = \{1, 2\}$

$\begin{matrix} 1 & 0 \\ 1 & -2 \end{matrix} \qquad A_{B_0}^{-1} \; \frac{1}{-2} \begin{vmatrix} -2 & -0 \\ -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -0 \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix}$

$x_0 = A_{B_0}^{-1} b_{B_0} = \begin{vmatrix} 1 & 0 \\ 1/2 & -1/2 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$

$y_0 = c\,A_{B_0}^{-1} = \begin{vmatrix} -1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 1/2 & -1/2 \end{vmatrix} = \begin{vmatrix} -1/2 & -1/2 \end{vmatrix}$

$h = 1$

$-A_{B_0}^{-1} = \begin{vmatrix} \boxed{\begin{matrix} -1 \\ -1/2 \end{matrix}} & \begin{matrix} 0 \\ 1/2 \end{matrix} \end{vmatrix}$

ξ

$A_N \xi = \begin{vmatrix} 0 & 1 \\ 1 & 1 \\ -4 & 2 \end{vmatrix} \cdot \begin{vmatrix} -1 \\ -1/2 \end{vmatrix} = \begin{vmatrix} -1/2 \\ -3/2 \\ 3 \end{vmatrix} \qquad K = 5$

$B_1 = \{2, 5\}$

$$A_{B_1} = \begin{vmatrix} 1 & -2 \\ -4 & 2 \end{vmatrix} \qquad A_{B_1}^{-1} = \frac{1}{\underbrace{2-8}_{-6}} \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} = \begin{vmatrix} -1/3 & -1/3 \\ -2/3 & -1/6 \end{vmatrix}$$

$$x_1 = A_{B_1}^{-1} b = \begin{vmatrix} -1/3 & -1/3 \\ -2/3 & -1/6 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \end{vmatrix} = \begin{vmatrix} -1 \\ -1 \end{vmatrix}$$

$$y_1 = c A_{B_1}^{-1} = [-1 \quad 1] \begin{vmatrix} -1/3 & -1/3 \\ -2/3 & -1/6 \end{vmatrix} = \begin{vmatrix} -1/3 & 1/6 \end{vmatrix}$$

$h = 2$

$$-A_{B_1}^{-1} = \begin{vmatrix} \boxed{\begin{matrix} 1/3 \\ 2/3 \end{matrix}} & \begin{matrix} 1/3 \\ 1/6 \end{matrix} \end{vmatrix} \qquad A_N \xi = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 1/3 \\ 2/3 \end{vmatrix} = \begin{vmatrix} 1/3 \\ 2/3 \\ 1 \end{vmatrix}$$

ξ

$+1$

$$\frac{b_i - A_i \bar{x}}{A_i \xi} \quad \begin{cases} 1: & 1 - (1 \ 0)\begin{pmatrix} -1 \\ -1 \end{pmatrix} \Big/ 1/3 = 6 \\ 3: & 1 - (0 \ 1)\begin{pmatrix} -1 \\ -1 \end{pmatrix} \Big/ 2/3 = \frac{2 \cdot 3}{2} = 3 \\ 4: & 2 - (1 \ 1)\begin{pmatrix} -1 \\ -1 \end{pmatrix} \Big/ 1 = 4 \end{cases}$$

$K = 3$

$B_2 = \{3, 5\}$

$$A_{B_2} = \begin{vmatrix} 0 & 1 \\ -4 & 2 \end{vmatrix} \qquad A_{B_2}^{-1} = \frac{1}{0+4} \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{-1}{4} \\ 1 & 0 \end{vmatrix}$$

$$x_2 = A_{B_2}^{-1} b_{B_2} = \begin{vmatrix} \frac{1}{2} & \frac{-1}{4} \\ 1 & 0 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

$$y_2 = c A_{B_2}^{-1} = \begin{vmatrix} -1 & 1 \end{vmatrix} \begin{vmatrix} 1/2 & -1/4 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 1/2 & 1/4 \end{vmatrix}$$

Esercizio 3.9. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\max \quad x_1 + 2x_2$$

$$\begin{aligned} x_1 &\le 1 \\ x_1 &\ge -1 \\ x_2 &\le -x_1 + 1 \\ -x_2 &\ge -x_1 - 1 \\ x_2 &\ge x_1 - 2 \\ x_1 + x_2 + 2 &\ge 0 \end{aligned}$$

Si parta dalla base ammissibile costituita dagli ultimi due vincoli.

$x = x_1$

$y = x_2$

$\max \; x + 2y \qquad c = [1 \;\; 2]$

1) $x \le 1$

2) $-x \le 1$

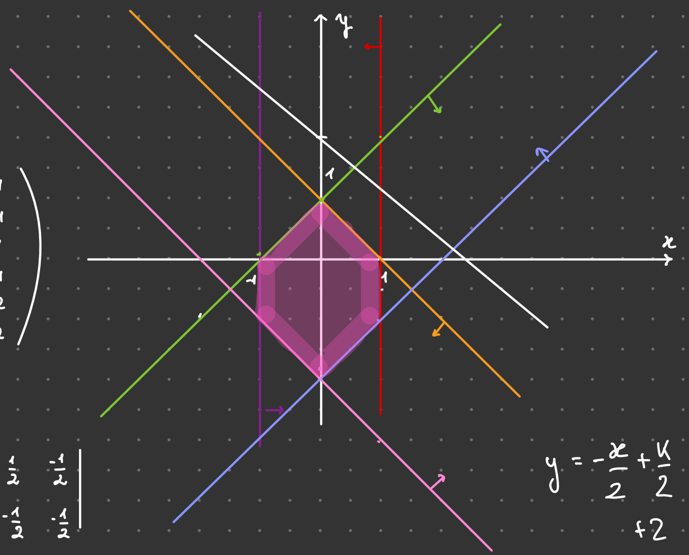
3) $x + y \le 1$

4) $-x + y \le 1$

5) $x - y \le 2$

6) $-x - y \le 2$

$$A = \begin{pmatrix} 1 & 0 \\ -1 & 0 \\ 1 & 1 \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}$$



$y = -\frac{x}{2} + \frac{k}{2}$

$+2$

x	
-1	

$B_0 = \{5, 6\}$

$$A_{B_0} = \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} \qquad A_{B_0}^{-1} = \frac{1}{-1-1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{vmatrix}$$

$$x_0 = A_{B_0}^{-1} b_{B_0} = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{vmatrix} \begin{vmatrix} 2 \\ 2 \end{vmatrix} = \begin{vmatrix} 0 \\ -2 \end{vmatrix}$$

$$y_0 = c A_{B_0}^{-1} = |1 \;\; 2| \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{vmatrix} = |\underbrace{-1/2}_{h=5} \;\; -3/2|$$

$$-A_{B_0}^{-1} = \begin{vmatrix} \underbrace{\boxed{\begin{matrix} -\frac{1}{2} \\ +\frac{1}{2} \end{matrix}}}_{\xi} & \begin{matrix} +\frac{1}{2} \\ +\frac{1}{2} \end{matrix} \end{vmatrix}$$

$$A_N \xi = \begin{vmatrix} 1 & 0 \\ -1 & 0 \\ 1 & 1 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} -1/2 \\ +1/2 \end{vmatrix} = \begin{vmatrix} -1/2 \\ 1/2 \\ 0 \\ 1 \end{vmatrix}$$

$$\min \frac{b_i - A_i x_0}{A_i \xi}$$

$i = 2$: $\dfrac{1 - (-1 \;\; 0)\begin{pmatrix} 0 \\ -2 \end{pmatrix}}{\frac{1}{2}} = 1 \cdot 2 = 2 \qquad K = 2$

$i = 4$: $\dfrac{1 - (-1 \;\; 1)\begin{pmatrix} 0 \\ -2 \end{pmatrix}}{1} = 1 + 2 = 3$

$B_1 \{2,6\}$

$$A_{B_1} = \begin{vmatrix} -1 & 0 \\ -1 & -1 \end{vmatrix} \qquad A^{-1}_{B_1} = \frac{1}{1-0} \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix}$$

$$x_1 = A^{-1}_{B_1} b_{B_1} = \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \end{vmatrix} = \begin{vmatrix} -1 \\ -1 \end{vmatrix} \qquad y_1 = c A^{-1}_{B_1} = \begin{vmatrix} 1 & 2 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & \underset{\downarrow\; h=6}{-2} \end{vmatrix}$$

$$-A^{-1}_{B_1} = \begin{vmatrix} 1 & \boxed{\begin{matrix} 0 \\ 1 \end{matrix}} \\ -1 & \end{vmatrix} \to \xi \qquad A_N \xi = \begin{vmatrix} 1 & 0 \\ 1 & 1 \\ -1 & 1 \\ 1 & -1 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ 1 \\ -1 \end{vmatrix}$$

$$\min \frac{b_i - A_i x}{A_N \xi} \quad \begin{cases} i=3: & \dfrac{1-(1\ 1)\begin{pmatrix} -1 \\ -1 \end{pmatrix}}{1} = 1+2 = 3 \\ i=4: & \dfrac{1-(-1\ 1)\begin{pmatrix} -1 \\ -1 \end{pmatrix}}{1} \;\; 1-1 \;\; = 1 \qquad k=4 \end{cases}$$

$B_2 = \{2,4\}$

$$A_{B_2} = \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} \qquad A^{-1}_{B_2} = \frac{1}{-1-0} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix}$$

$$x_2 = \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} -1 \\ 0 \end{vmatrix} \qquad y_2 = c A^{-1}_{B_2} = \begin{vmatrix} 1 & 2 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} = \begin{vmatrix} \underset{\downarrow\; h=2}{-3} & 2 \end{vmatrix}$$

$$-A^{-1}_{B_2} = \begin{vmatrix} \boxed{\begin{matrix} 1 \\ 1 \end{matrix}} & \begin{matrix} 0 \\ -1 \end{matrix} \end{vmatrix} \to \xi \qquad A_N \xi = \begin{vmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \\ -1 & -1 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 0 \\ -2 \end{vmatrix} \begin{matrix} 1 \\ 3 \\ 5 \\ 6 \end{matrix}$$

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{pmatrix} 1 & 0 \\ \cancel{-1} & \cancel{0} \\ 1 & 1 \\ \cancel{-1} & \cancel{1} \\ 1 & -1 \\ -1 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\min \frac{b_i - A_i x}{A_i \xi} \quad \begin{cases} i=1: & \dfrac{1-(1\ 0)\begin{pmatrix} -1 \\ 0 \end{pmatrix}}{1} = 1+1 = 2 \\ i=3: & \dfrac{1-(1\ 1)\begin{pmatrix} -1 \\ 0 \end{pmatrix}}{2} \;\; \dfrac{2}{2} = 1 \qquad K=3 \end{cases}$$

$B_3 = \{3,4\}$

$$A_{B_3} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \qquad A^{-1}_{B_3} = \frac{1}{1+1} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{vmatrix}$$

$$x_3 = \begin{vmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \qquad y_3 = \begin{vmatrix} 1 & 2 \end{vmatrix} \begin{vmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{vmatrix} = \begin{vmatrix} \frac{3}{2} & \frac{1}{2} \end{vmatrix}$$

x_3 e y_3 soluzioni ottime e il valore ottimo è $c x_3 = \begin{vmatrix} 1 & 2 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = 2$

Esercizio 3.30. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\max x + y + 2$$

$$y + 2 \geq 2$$
$$x - 3 \geq -3$$
$$y + x - 4 \leq 0$$

Si parta dalla base ammissibile corrispondente ai primi due vincoli.

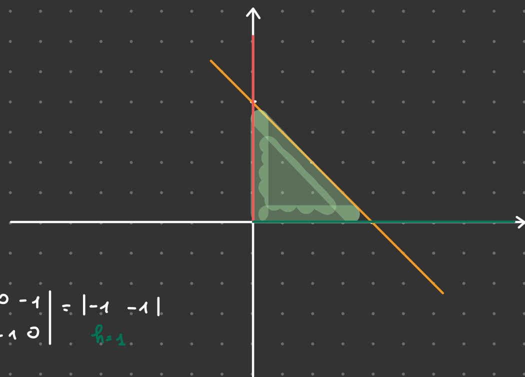
$\max \; x+y+2$

1) $-y \leq 0$

2) $-x \leq 0$

3) $y+x \leq 4$

$A = \begin{vmatrix} 0 & -1 \\ -1 & 0 \\ 1 & 1 \end{vmatrix} \quad b = \begin{vmatrix} 0 \\ 0 \\ 4 \end{vmatrix}$



1ª Iterazione

$B_0 = \{1, 2\}$

$A = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \qquad A_0^{-1} = \frac{1}{-1} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$x_0 = A_0^{-1} b_0 = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix} \qquad y_0 = c\, A_0^{-1} = \begin{vmatrix} 1 & 1 \end{vmatrix} \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & -1 \end{vmatrix}$ $h=1$

$-A_0^{-1} = \begin{vmatrix} \boxed{\begin{matrix} 0 \\ 1 \end{matrix}} & \begin{matrix} 1 \\ 0 \end{matrix} \end{vmatrix}$ ξ_1

$A_N \xi = \begin{vmatrix} 1 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = 1$ $k=3$

2° Iterazione

$B_1 = \{2, 3\}$

$A_1 = \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} \qquad A_1^{-1} = \frac{1}{-1} \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}$

$x_1 = A_1^{-1} b_1 = \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 4 \end{vmatrix} = \begin{vmatrix} 0 \\ 4 \end{vmatrix} \qquad y_1 = \begin{vmatrix} 1 & 1 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \end{vmatrix}$

restituisce x_1 e y_1 come soluzioni

valore ottimo $cx + 2 = 6$

Esercizio 3.36. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\min x_1 + 4x_2$$

$$\begin{aligned} x_1 &\le 4 \\ x_2 &\le 4 \\ x_2 + 1 - x_1 &\ge 0 \\ x_2 - x_1 - 1 &\le 0 \\ x_2 + 3 &\ge 0 \\ x_1 + 3 &\ge 0 \end{aligned}$$

Si parta dalla base ammissibile corrispondente ai primi due vincoli.

$x_1 = x$

$x_2 = y$

$\max \ -x - 4y$

1) $x \le 4$

2) $y \le 4$

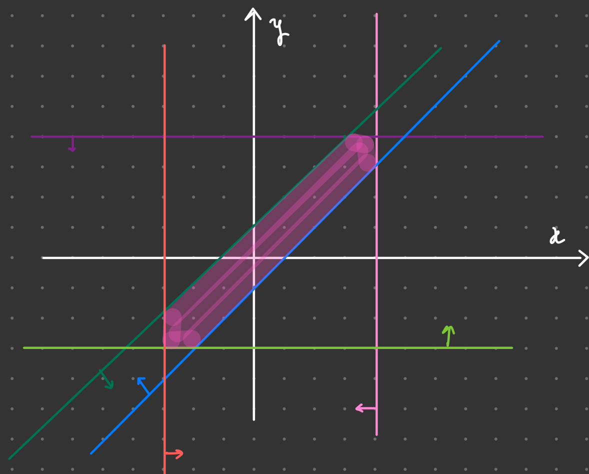
3) $x - y \le 1$

4) $-x + y \le 1$

5) $-y \le 3$

6) $-x \le 3$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ -1 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 4 \\ 1 \\ 1 \\ 3 \\ 3 \end{pmatrix}$$



prima iterazione

$B_0 \{1, 2\}$

$$A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad A^{-1}_{B_0} = \frac{1}{1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$x_{B_0} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 4 \\ 4 \end{vmatrix} = \begin{vmatrix} 4 \\ 4 \end{vmatrix} \qquad y_{B_0} = c A^{-1}_{B_0} = \begin{vmatrix} -1 & -4 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \underset{h=1}{-1} & -4 \end{vmatrix}$$

$$-A^{-1}_{B_0} = \begin{vmatrix} \boxed{\begin{matrix} -1 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ -1 \end{matrix} \end{vmatrix} \quad \xi \qquad A_N \xi = \begin{vmatrix} 1 & -1 \\ -1 & 1 \\ 0 & -1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} -1 \\ 0 \end{vmatrix} = \begin{array}{|c|c} -1 & 3 \\ 1 & 4 \\ 0 & 5 \\ 1 & 6 \end{array}$$

$$\frac{b_i - A_i x}{A_i \xi} \begin{cases} i=4: & \dfrac{1 - (-1\ 1)\begin{pmatrix} 4 \\ 4 \end{pmatrix}}{1} = 1 \\ i=6: & \dfrac{3 - (-1\ 0)\begin{pmatrix} 4 \\ 4 \end{pmatrix}}{1} = 7 \end{cases} \qquad K = 4$$

iterazione

$B_1 = \{2, 4\}$

$$A_{B_1} = \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} \qquad A^{-1}_{B_1} = \frac{1}{0+1} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} \qquad x_{B_1} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} 4 \\ 1 \end{vmatrix} = \begin{vmatrix} 3 \\ 4 \end{vmatrix}$$

$$y_{B_1} = c A^{-1}_{B_1} = \begin{vmatrix} -1 & -4 \end{vmatrix} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} \underset{h=2}{-5} & 1 \end{vmatrix} \qquad -A^{-1}_{B_1} = \begin{vmatrix} \boxed{\begin{matrix} -1 \\ -1 \end{matrix}} & \begin{matrix} 1 \\ 0 \end{matrix} \end{vmatrix} \quad \xi \qquad A_N \xi = \begin{vmatrix} 1 & 0 \\ 1 & -1 \\ 0 & -1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} -1 \\ -1 \end{vmatrix} = \begin{array}{|c|c} -1 & 1 \\ 0 & 3 \\ 1 & 5 \\ 1 & 6 \end{array}$$

seconda

$$\frac{b_i - A_i x}{A_i \xi} \quad \begin{cases} i=5: & \dfrac{3+4}{1} = 7 \\[2ex] i=6: & \dfrac{3+3}{1} = 6 \quad k=6 \end{cases}$$

Terza iterazione

$B_2 = \{4, 6\}$

$$A_{B_2} = \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} \qquad A^{-1}_{B_2} = \begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix} \qquad x_{B_2} = \begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix} \begin{vmatrix} 1 \\ 3 \end{vmatrix} = \begin{vmatrix} -3 \\ -2 \end{vmatrix} \qquad y_{B_2} = c A^{-1}_{B_2} = \begin{vmatrix} -1 & -4 \end{vmatrix} \begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix} = \begin{vmatrix} -4 & 5 \end{vmatrix}$$

$h=4$

$$-A^{-1}_{B_2} = \begin{vmatrix} \boxed{\begin{matrix} 0 \\ -1 \end{matrix}} & \begin{matrix} 1 \\ 1 \end{matrix} \end{vmatrix} \qquad \xi$$

$$A_N \xi = \begin{vmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 0 & -1 \end{vmatrix} \begin{vmatrix} 0 \\ -1 \end{vmatrix} = \begin{array}{|c|c} 0 & 1 \\ -1 & 2 \\ 1 & 3 \\ 1 & 5 \end{array}$$

$$\frac{b_i - A_i x}{A_i \xi} \quad \begin{cases} i=3: & \dfrac{1+5}{1} = 6 \\[2ex] i=5: & \dfrac{3+2}{1} = 5 \end{cases}$$

quarta iterazione

$B_3 = \{5, 6\}$

$$A_{B_3} = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \qquad A^{-1}_{B_3} = \frac{1}{-1} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \qquad x_3 = \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} 3 \\ 3 \end{vmatrix} = \begin{vmatrix} -3 \\ -3 \end{vmatrix} \qquad y_3 = \begin{vmatrix} -1 & -4 \end{vmatrix} \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} 4 & 1 \end{vmatrix}$$

Valore ottimo

$$c x_3 = \begin{vmatrix} -1 & -4 \end{vmatrix} \begin{vmatrix} -3 \\ -3 \end{vmatrix} = 3 + 12 = 15$$

3.2.8 Temi d'esame 2020

Esercizio 3.36. Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\min -2x$$

$$\begin{aligned} y &\geq 0 \\ 3x - y &\geq 0 \\ -x + 10 &\geq 3y \\ 2x - 6 &\leq y \end{aligned}$$

Si parta dalla base ammissibile corrispondente ai primi due vincoli. $[x = (4,2)]$

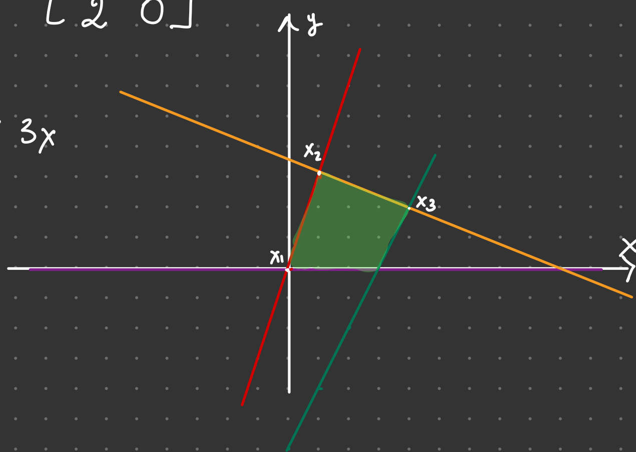
$\max \quad 2x \qquad [2 \;\; 0]$

1 $-y \leq 0$

2 $-3x + y \leq 0 \qquad y \leq 3x$

3 $x + 3y \leq 10$

4 $2x - y \leq 6$



$$A = \begin{bmatrix} 0 & -1 \\ -3 & 1 \\ 1 & 3 \\ 2 & -1 \end{bmatrix} \qquad b \begin{bmatrix} 0 \\ 0 \\ 10 \\ 6 \end{bmatrix}$$

$B\{1,2\}$

$$A_1 = \begin{bmatrix} 0 & -1 \\ -3 & 1 \end{bmatrix} \quad A_1^{-1} = \begin{bmatrix} -1/3 & -1/3 \\ -1 & 0 \end{bmatrix} \qquad \bar{x}_1 = A_1^{-1} b_1 = \begin{bmatrix} -1/3 & -1/3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y = c A_1^{-1} = [2 \;\; 0] \begin{bmatrix} -1/3 & -1/3 \\ -1 & 0 \end{bmatrix} = \left[\underset{h=1}{-\tfrac{2}{3}} \;\; -\tfrac{2}{3} \right]$$

$$-A_1^{-1} = \begin{bmatrix} \boxed{\begin{matrix} +1/3 \\ +1 \end{matrix}} & \begin{matrix} +1/3 \\ 0 \end{matrix} \end{bmatrix} \quad \xi$$

$$A_N \xi = \begin{array}{c} 3 \\ 4 \end{array} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ -\frac{1}{3} \end{bmatrix} \qquad K=3$$

$B_2 \{2,3\}$

$$A_2 = \begin{bmatrix} -3 & 1 \\ 1 & 3 \end{bmatrix} \qquad A_2^{-1} = \begin{bmatrix} \frac{-3}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{3}{10} \end{bmatrix} \qquad \bar{x}_2 = \begin{bmatrix} \frac{-3}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} 0 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\bar{y} = cA_2^{-1} = [2 \;\; 0] \begin{bmatrix} -3/10 & 1/10 \\ 1/10 & 3/10 \end{bmatrix} = [\underset{h=2}{\frac{-3}{5}} \;\; \frac{1}{5}]$$

$$-A_2^{-1} = \begin{bmatrix} \boxed{\begin{matrix} 3/10 \\ -1/10 \end{matrix}} & \begin{matrix} -1/10 \\ -3/10 \end{matrix} \end{bmatrix} \qquad A_N \xi_{u}^{1} = \begin{bmatrix} 0 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3/10 \\ -1/10 \end{bmatrix} = \begin{bmatrix} 1/10 \\ 7/10 \end{bmatrix}$$

ξ

$$\frac{b_i - A_i \bar{x}_2}{A_i \xi} = \begin{cases} i=1: & \dfrac{0 - [0 \; -1] \begin{bmatrix} 1 \\ 3 \end{bmatrix}}{1/10} = 0 + 3 \cdot 10 = 30 \\[2ex] i=4: & \dfrac{6 - [2 \; -1] \begin{bmatrix} 1 \\ 3 \end{bmatrix}}{7/10} = \dfrac{70}{7} = 10 \end{cases} \qquad k = 4$$

$B_3 \{3,4\}$

$$A_3 = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \qquad A_3^{-1} = \begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{bmatrix} \qquad \bar{x}_3 = A_3^{-1} b_3 \begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{bmatrix} \begin{bmatrix} 10 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$y_3 = c A_3^{-1} = [2 \;\; 0] \begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{bmatrix} = [2/7 \;\; 6/7]$$

$\bar{x}_3$ e $\bar{y}_3$ sono le soluzioni ottime

Branch and Bound

ESERCIZIO 1

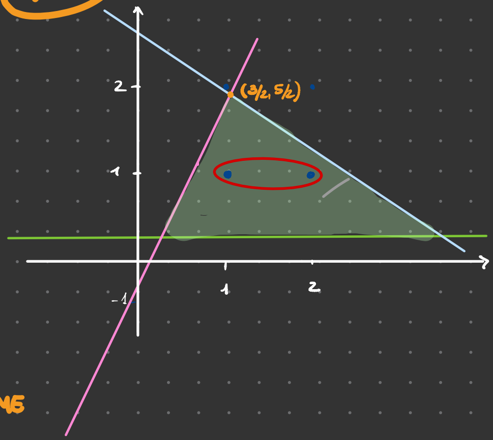
F.O.: $\min -x_1 - x_2$

vincoli:

$x_2 \geq 1/2$

$x_2 \leq -\frac{1}{2} + 2x_1$ $\qquad x_1, x_2 \in \mathbb{Z}$

$x_2 \leq -2x_1 + \frac{11}{2}$



1. CERCO CON IL SIMPLESSO LE SOLUZIONI OTTIME

(guardo il grafico) $\quad \max x_1 + x_2$

1) $-x_2 \leq -1/2$
2) $-2x_1 + x_2 \leq -1/2$
3) $2x_1 + x_2 \leq \frac{11}{2}$

$B_0 \{2,3\}$

$A_0 = \begin{bmatrix} -2 & 1 \\ 2 & 1 \end{bmatrix} \quad A_0^{-1} = \begin{bmatrix} -1/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} \quad \overline{x}_0 = \begin{bmatrix} -1/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} -1/2 \\ 11/2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 5/2 \end{bmatrix}$

$\overline{y}_0 = [1 \quad 1] \begin{bmatrix} -1/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = [1/4 \quad 3/4]$ $\qquad \overline{x}_0$ e $\overline{y}_0$ sono soluzioni ottime

2. CALCOLO IL VALORE OTTIMO

$-x_1 - x_2 \longrightarrow -\frac{3}{2} - \frac{5}{2} = -\cancel{\frac{8}{2}}^{4} = \boxed{-4}$

(meglio di così non possiamo fare)

3. PARTIZIONIAMO LA REGIONE AMMISSIBILE DEL PROBLEMA P^0 OTTENENDO DUE PROBLEMI P^1 E P^2.

$$P_0 \left[\begin{array}{l} \min -x_1 - x_2 \\ x_2 \geq 1/2 \\ x_2 \leq -1/2 + 2x_1 \\ x_2 \leq 2x_1 + 11/2 \end{array}\right.$$

P_1: $x_1 \leq 1$ P_2: $x_1 \geq 2$

questo perché

$$2 \leq x_1 \leq 1$$

$$\bar{x}_0 = \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$4 \leq \cancel{x} \leq 3$$

3.75

RISOLVO IL PROBLEMA P_1

$$A \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ 1 \end{bmatrix}$$

$B_0 = \{2, 4\}$

$$A_0 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \quad A_0^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\bar{x}_0^1 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{2} \end{bmatrix}$$

$$\bar{y}_0^1 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \end{bmatrix}$$

RISOLVO IL PROBLEMA P_2

$$A \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ -1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ -2 \end{bmatrix}$$

$B_0 = \{3, 4\}$

$$A_0 = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \quad A_0^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\bar{x}_0^2 = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 11/2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3/2 \end{bmatrix}$$

$$\bar{y}_0^2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

4. SCELGO QUALE PROBLEMA RILASSARE. PROCEDO CON $P_1 \rightarrow P_3$ e P_4

RISOLVO P_3

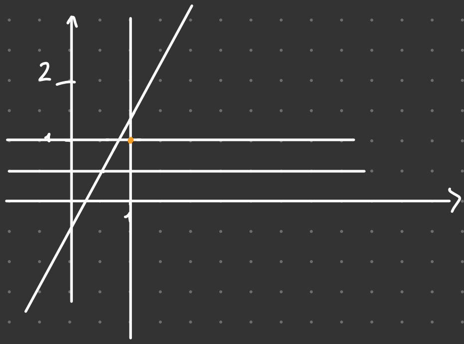
$$A \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ 1 \\ 1 \end{bmatrix}$$

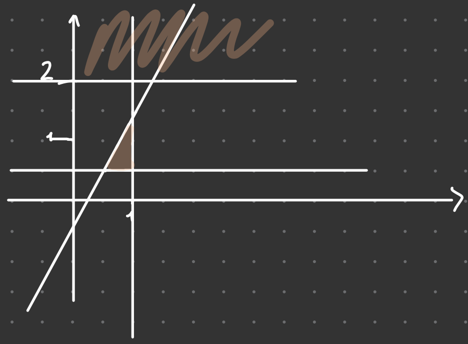
$B_0 \{2, 5\}$

RISOLVO P_4

$$A \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ 1 \\ -2 \end{bmatrix}$$

VUOTO





$B = \{4, 5\}$

$A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\bar{x} = A^{-1} b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

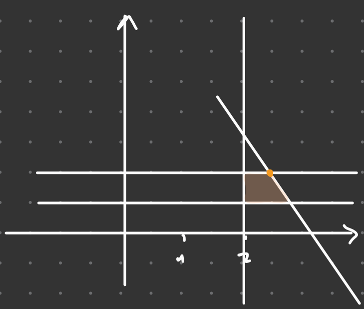
$z = -2$

VUOTO

5. DECIDO DI RILASSARE ANCHE P_2 IN QUANTO HA UNA Z CHE SI AVVICINA DI PIÙ A QUELLA OTTIMA

RISOLVO P_5

$A \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \\ \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ -2 \\ 1 \end{bmatrix}$

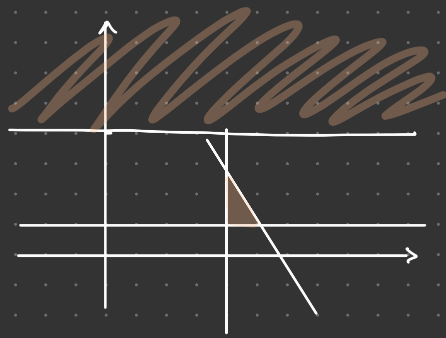


$B \{3, 5\}$

$A \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \quad A^{-1} \begin{bmatrix} 1/2 & -1/2 \\ 0 & 1 \end{bmatrix}$

RISOLVO P_6

$A \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \\ \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ -2 \\ 2 \end{bmatrix}$



VUOTO

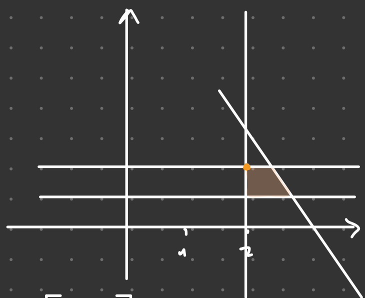
$$x = \begin{bmatrix} 1/2 & -1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 11/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9/4 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \end{bmatrix}$$

6. RILASSO IL PROBLEMA P_5 IN P_7 E P_8

RISOLVO P_7

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ -1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ -2 \\ 1 \\ 2 \end{bmatrix}$$



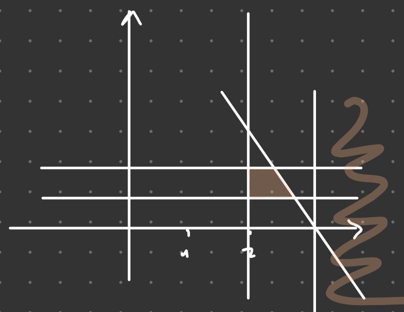
$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y = [1 \quad 1]$$

RISOLVO P_8

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \\ \end{array} \begin{bmatrix} 0 & -1 \\ -2 & 1 \\ 2 & 1 \\ -1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} -1/2 \\ -1/2 \\ 11/2 \\ -2 \\ 1 \\ 3 \end{bmatrix}$$



VUOTO

P_0 $\quad x_0 = \left(\frac{3}{2}, \frac{5}{2}\right)$ $\quad z_0 = -4$

$x_1 \le 1$ → P_1 $\qquad x_1 \ge 2$ → P_2

$\overline{x_1} = \left(1, \frac{3}{2}\right)$ $\quad z_1 = -\frac{5}{2}$ $\quad P_1$

P_2 $\quad \overline{x_2} = \left(2, \frac{3}{2}\right)$ $\quad z_2 = -2 - \frac{3}{2} = \frac{-4-3}{2} = -\frac{7}{2}$

P_1: $x_2 \le 1$ → P_3 $\qquad x_2 \ge 2$ → P_4

P_2: $1 \le x_2$ → P_5 $\qquad x_2 < 2$ → P_6

$2 \le x_2 \le 1$

$\overline{x_3} = (1, 1)$

$z_3 = -2$

P_3 $\qquad P_4$ VUOTO $\qquad \left(\frac{9}{4}, 1\right)$ P_5 $z = -\frac{13}{4}$ $\qquad P_6$ VUOTO

P_5: $x_1 \le 2$ → P_7 $\qquad x_1 \ge 3$ → P_8

LO SCARTIAMO

$z_3 < z_7$

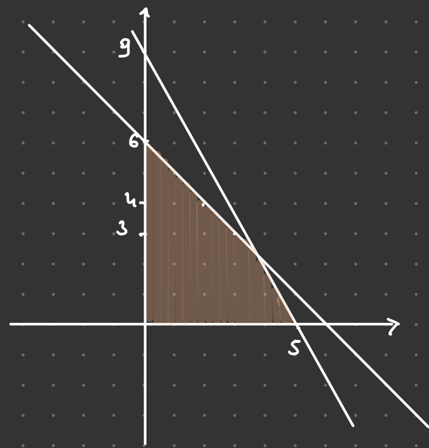
$x(2\ 1)$ $\quad z_7 = -3$ $\qquad P_7$ $\qquad P_8$ VUOTO

↳ SI AVVICINA DI PIÙ ALLA SOLUZIONE

ESERCIZIO 2

$\max \quad 8x+5y$

1 $\quad x+y \le 6$
2 $\quad 9x+5y \le 45$
$\quad x, y \in \mathbb{Z}$
4 $\quad x \ge 0 \; \cdots \to \; -x \le 0$
5 $\quad y \ge 0 \qquad -y \le 0$



$B \{1, 2\}$

$$A \begin{bmatrix} 1 & 1 \\ 9 & 5 \end{bmatrix} \qquad A^{-1} \begin{bmatrix} -5/4 & 1/4 \\ 9/4 & -1/4 \end{bmatrix}$$

$$x = \begin{bmatrix} -5/4 & 1/4 \\ 9/4 & -1/4 \end{bmatrix} \begin{bmatrix} 6 \\ 45 \end{bmatrix} = \begin{bmatrix} 15/4 \\ 9/4 \end{bmatrix} \qquad y = \begin{bmatrix} 8 & 5 \end{bmatrix} \begin{bmatrix} -5/4 & 1/4 \\ 9/4 & -1/4 \end{bmatrix} = \begin{bmatrix} 5/4 & 3/4 \end{bmatrix}$$

$$Z = \cancel{8}^{\,2} \frac{15}{\cancel{4}} + 5 \frac{9}{4}$$
$$= 30 + \frac{45}{4} = \frac{120 + 45}{4} = \frac{185}{4}$$

$P_0 \quad x_0 = \left(\frac{15}{4}, \frac{9}{4} \right) \quad z = \frac{185}{4}$

$x \le 3$ → P_1 ; $x \ge 4$ → P_2

$x_1 (3, 3) \quad z = 39 \quad P_1$

$P_2 \quad x_2 \left(4, \frac{9}{5} \right) \quad z_2 = 41$

$y \le 1$ → P_3 ; $y \ge 2$ → P_4

$x_3 \left(\frac{40}{9}, 1 \right) \quad z_3 = \frac{365}{9} \quad P_3$

P_4
VUOTO

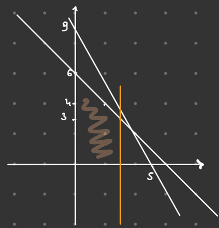
$x \le 4$ → P_5 ; $x \ge 5$ → P_6

$x_5 (4, 1) \quad z_3 = 37 \quad P_5$

$P_6 \quad x_6 = (5, 0) \quad z_6 = 40$

RISOLVO P1

$$A = \begin{bmatrix} 1 & 1 \\ 9 & 5 \\ -1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix} \quad b \begin{bmatrix} 6 \\ 45 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$



$B\{1,5\}$

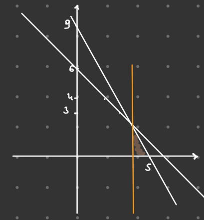
$$x = \overset{A^{-1}}{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}} \overset{b}{\begin{bmatrix} 6 \\ 3 \end{bmatrix}} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\bar{y} = \begin{bmatrix} 8 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \end{bmatrix}$$

$$z = 24 + 15 = 39$$

RISOLVO P2

$$A = \begin{bmatrix} 1 & 1 \\ 9 & 5 \\ -1 & 0 \\ 0 & -1 \\ -1 & 0 \end{bmatrix} \quad b \begin{bmatrix} 6 \\ 45 \\ 0 \\ 0 \\ -4 \end{bmatrix}$$



$B\{2,5\}$

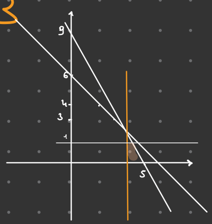
$$x = \begin{bmatrix} 0 & -1 \\ \frac{1}{5} & \frac{9}{5} \end{bmatrix} \begin{bmatrix} 45 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 9/5 \end{bmatrix}$$

$$y = \begin{bmatrix} 8 & 5 \end{bmatrix} \overset{A^{-1}}{\begin{bmatrix} 0 & -1 \\ \frac{1}{5} & \frac{9}{5} \end{bmatrix}} = \begin{bmatrix} \frac{8}{5} & 1 \end{bmatrix}$$

$$z = 8 \cdot 4 + 5 \cdot \frac{9}{5} = 41$$

RISOLVO P3

$$A = \begin{bmatrix} 1 & 1 \\ 9 & 5 \\ -1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 6 \\ 45 \\ 0 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$



$B\{2,6\}$

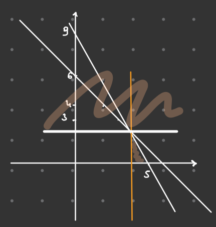
$$x = \overset{A^{-1}}{\begin{bmatrix} 1/9 & -5/9 \\ 0 & 1 \end{bmatrix}} \overset{b}{\begin{bmatrix} 45 \\ 1 \end{bmatrix}} = \begin{bmatrix} 40/9 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 8 & 5 \end{bmatrix} \begin{bmatrix} 1/9 & -5/9 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8/9 & \frac{5}{9} \end{bmatrix}$$

$$z = 8 \, \frac{40}{9} + 5 = \frac{320}{9} + \frac{45}{9} = \frac{365}{9}$$

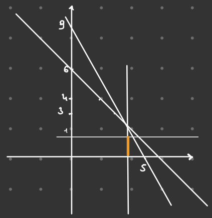
RISOLVO P4

PROBLEMA VUOTO



RISOLVO P5

$$A=\begin{bmatrix}1 & 1\\ 9 & 5\\ -1 & 0\\ 0 & -1\\ -1 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix}\quad b=\begin{bmatrix}6\\ 45\\ 0\\ 0\\ -4\\ 1\\ 4\end{bmatrix}$$



$B\{6,7\}$

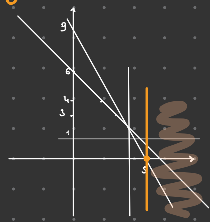
$$x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 4\end{bmatrix}=\begin{bmatrix}4\\ 1\end{bmatrix}$$

$$y=\begin{bmatrix}8 & 5\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}5 & 8\end{bmatrix}$$

$$z = 32+5 = 37$$

RISOLVO P6

$$A=\begin{bmatrix}1 & 1\\ 9 & 5\\ -1 & 0\\ 0 & -1\\ -1 & 0\\ 0 & 1\\ -1 & 0\end{bmatrix}\quad b=\begin{bmatrix}6\\ 45\\ 0\\ 0\\ -4\\ 1\\ -5\end{bmatrix}$$



$$x=\begin{bmatrix}0 & -1\\ \frac{1}{5} & \frac{9}{5}\end{bmatrix}\begin{bmatrix}45\\ -5\end{bmatrix}=\begin{bmatrix}5\\ 0\end{bmatrix}$$

$$y=\begin{bmatrix}8 & 5\end{bmatrix}\begin{bmatrix}0 & -1\\ \frac{1}{5} & \frac{9}{5}\end{bmatrix}=\begin{bmatrix}1 & 1\end{bmatrix}$$

$$z = 8\cdot 5 = 40$$

LA SOLUZIONE CHE SI AVVICINA DI PIÙ A P_0 E P_5