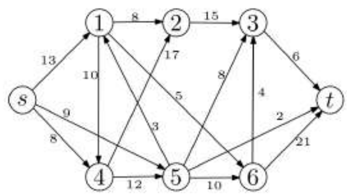
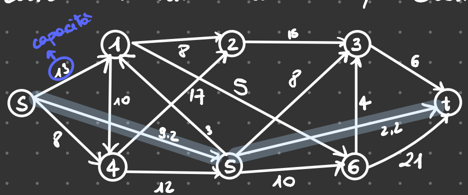
Reti di flusso

2.1 Esercizi con soluzioni

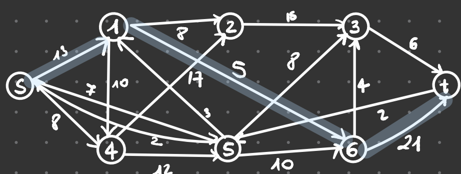
Esercizio 2.1. Si trovi il flusso massimo (e il taglio di capacità minima) nella rete seguente.



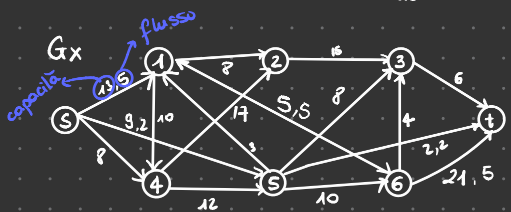
EK → cerchiamo i cammini aumentati sul grafo residuo visitando i cammini più brevi che vanno da s a t.



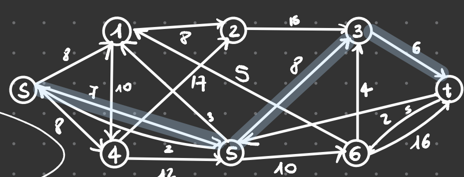
min {9, 2} = 2



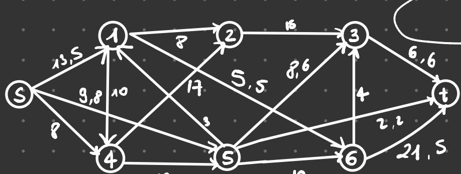
min {13, 5, 21} = 5

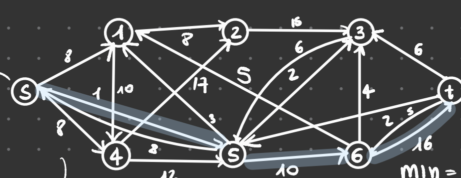


FLUSSO

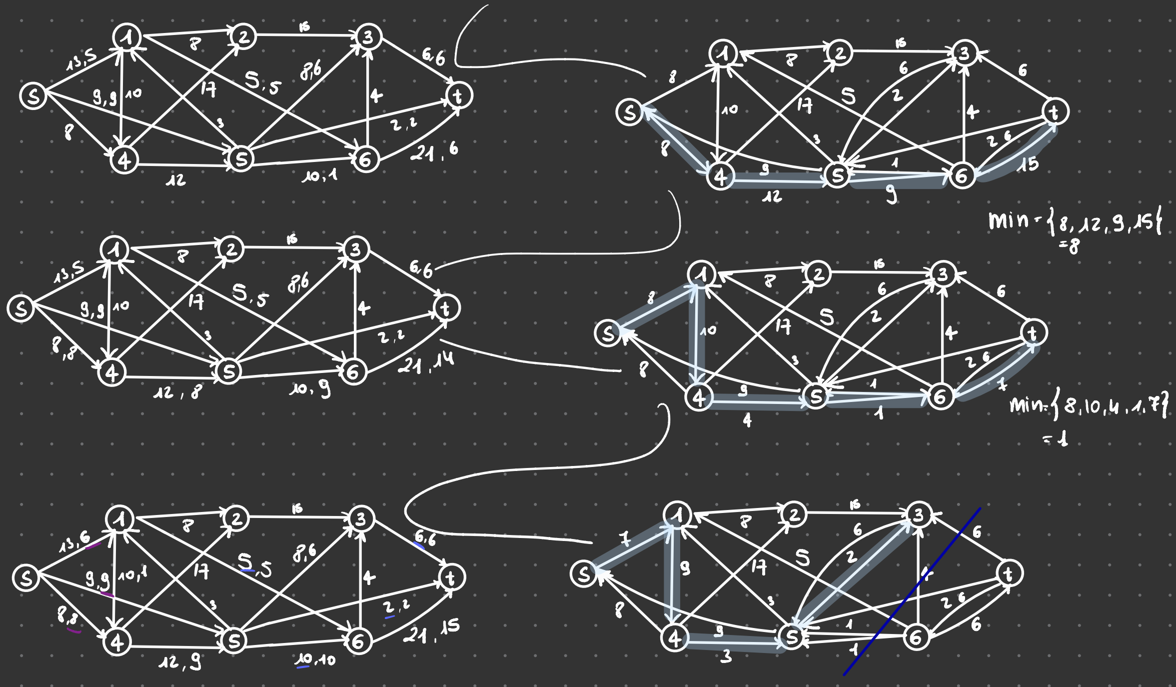


min = {7, 8, 6} = 6





min = {1, 10, 16} = 1



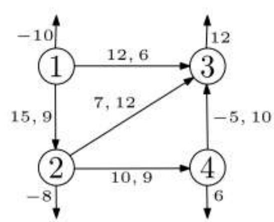
TEOREMA MAX-FLOW MIN CUT

5+10+2+6 = 23

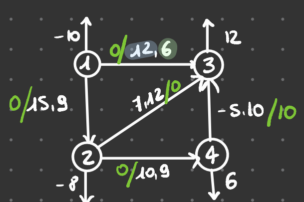
23 = 6+9+8

il valore del flusso è massimo = capacità del taglio è minima

Esercizio 2.2. Si risolva il seguente problema MCF tramite l'algoritmo dei cammini minimi successivi.



pseudoflusso minimale iniziale : costo > 0 → FLUSSO = 0
costo < 0 → FLUSSO = capacità



costo
capacità
pseudoflusso = x

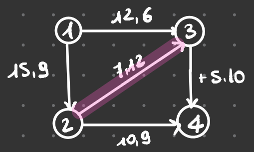
Ora devo trasformare il pseudoflusso in flusso
↓
lo faccio cercando il cammino di costo minimo che vada da un nodo pos a un nodo neg.

Quindi prima bisogna controllare gli sbilanciamenti che siano $\neq 0$

in 1 → $+10 - 0 > 0$
in 2 → $8 - 0 > 0$
in 3 → $10 - 12 = -2 < 0$
4 → $0 - 16 < 0$

+	−
1	3
2	4

1° cammino di costo minimo che va dal nodo pos al nodo negativo.



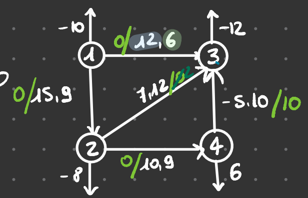
poi bisogna fare il

$\min \{ \text{sbilanc.}(2), \text{ capacità del cammino scelto}, -\text{sbilanc.}(3) \}$

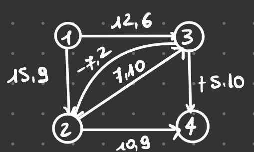
nodo positivo — nodo negativo

$\min = \{8, 12, 2\} = 2$

Ora pompo di 2 unità lo pseudoflusso



dopo di che mi ricostruisco il grafo

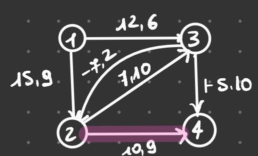


- non ho saturato l'arco, quindi costruisco il discorde con costo con segno opposto e la capacità = all'unità di flusso che ci abbiamo fatto "scorrere"
- modifico anche la capacità dell'arco concorde che diminuisce dell'unità fatta scorrere.

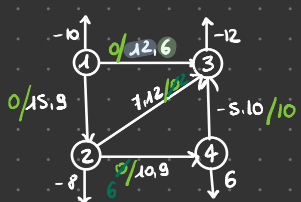
Ora si ricontrolla $g(x)$. Poi cerchiamo il camino di costo min. da un nodo positivo a un nodo negativo

$1 = 10 > 0$
$2 = 8 - 2 = 6 > 0$
$3 = 10 + 2 - 12 = 0$
$4 = -10 - 6 = -16$

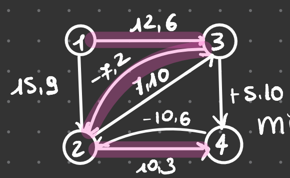
+	−
1	4
2	



$\min \{6, 9, 16\} = 6$

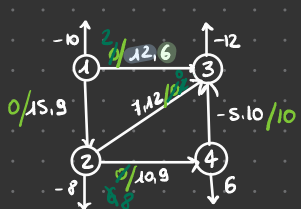


+	−
1	4

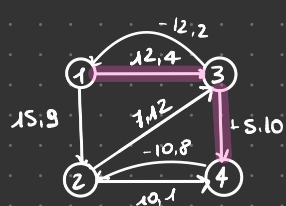


1 - 3 - 2 - 4 = 15 costo
1 - 2 - 4 = 25 costo

$\min = \{10, 6, 2, 3, 10\}$
$= 2$

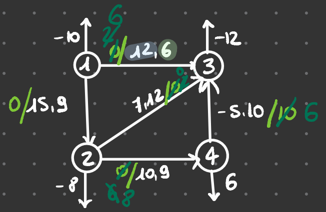


+	−
1	4

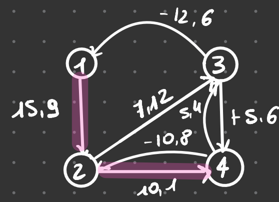


$\min = \{8, 4, 10, 8\}$

$\min = 4$

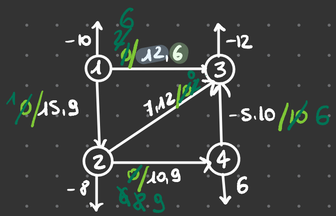


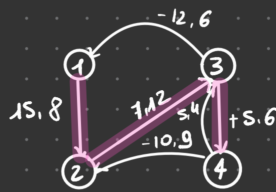
+	-
1	4



$\min\{4, 9, 1, 4\} = 1$

1-2-4 = 25

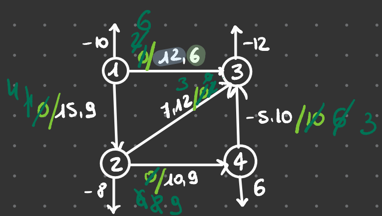




1-2-3-4?

15+7-5 = 17

$\min = \{3, 8, 12, 6, 3\} = 3$



controlliamo g(x)

nodo 1 → 10 − 6 − 4 = 0

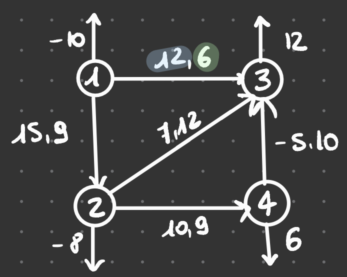
nodo 2 → 8 + 4 − 3 − 9 = 0

nodo 3 → 3 + 6 + 3 − 12 = 0

nodo 4 → 9 − 3 − 9 = 0

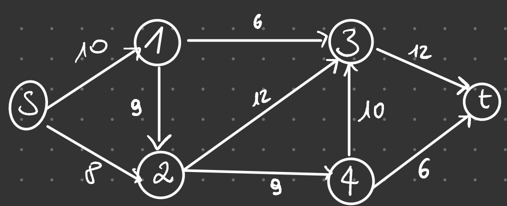
Esercizio 2.3. Si trovi il flusso di costo minimo per la rete dell'esercizio 2.2 tramite l'algoritmo di cancellazione dei cicli.

costo
capacità



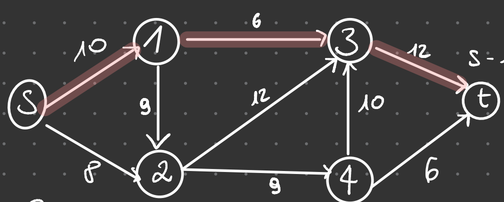
EDMONDS KARP
↓
FLUSSO MASSIMO
↓
[TEOREMA MAX-FLOW MIN CUT
↓
MASSIMO FLUSSO = MINIMA CAPACITÀ DEI TAGLI]
↓
CONTROLLO SE IL FLUSSO TROVATO È AMMISSIBILE
↓
CANCELLAZIONE DEI CICLI
↓
COSTO MINIMO

Trasformiamo G in G' → 1ª sorgente e un pozzo



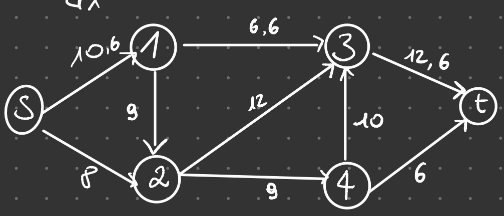
Ora calcoliamo il flusso massimo tramite l'algoritmo di E-K

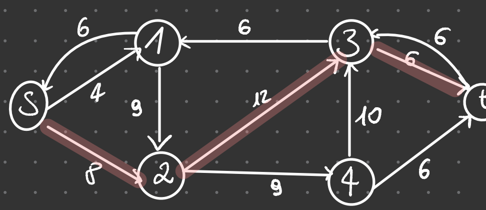
bisogna trovare i cammini aumentanti sul grafo residuo visitando solamente i cammini più brevi.



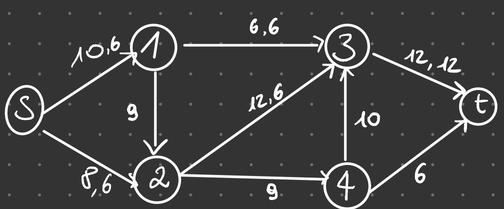
S-1-3-t
min{10, 6, 12} = 6

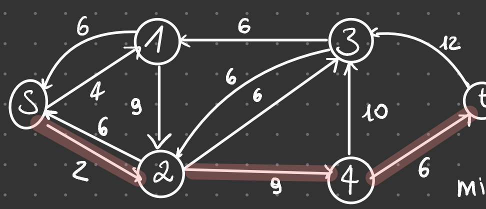
Gx



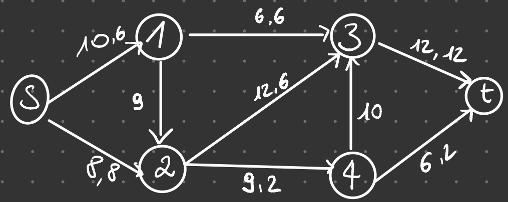


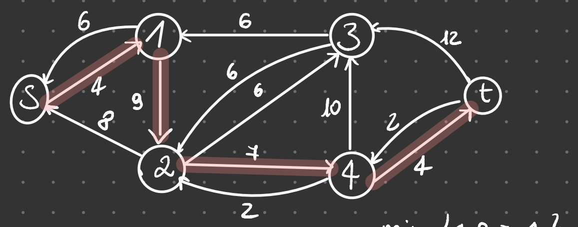
min{8, 12, 6} = 6



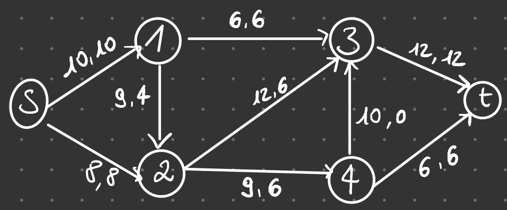


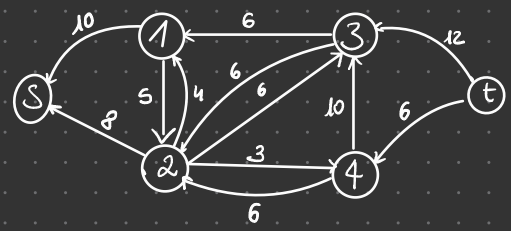
min = {2, 9, 6} = 2





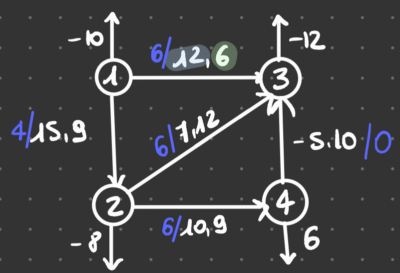
min = {4, 9, 7, 4} = 4





FLUSSO MASSIMO → non esistono più cammini aumentanti che vanno da s a t.

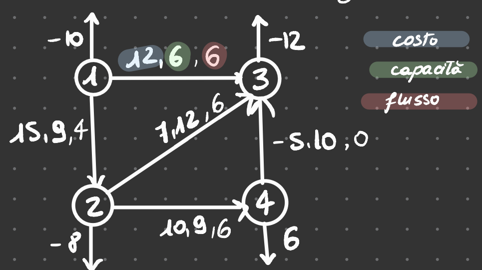
Ora controlliamo se il flusso massimo trovato è ammissibile



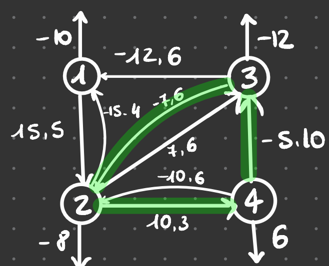
controlliamo gli sbilanciamenti

nodo 1 → 10 − 6 − 4 = 0
nodo 2 → 8 + 4 − 6 − 6 = 0
nodo 3 → 6 + 6 − 12 = 0
nodo 4 → 6 − 6 = 0

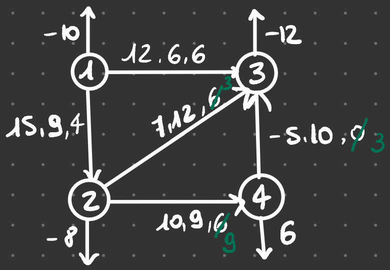
Procediamo con l'algoritmo di cancellazione dei cicli

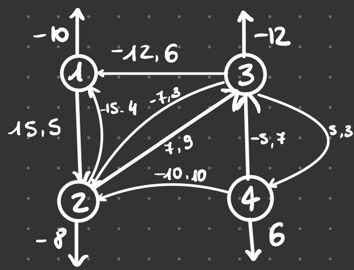


bisogna cercare i cicli di costo negativo



min = {6, 3, 10} = 3

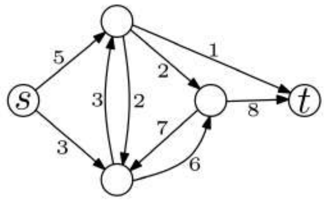




non esistono più cicli di costo minimo

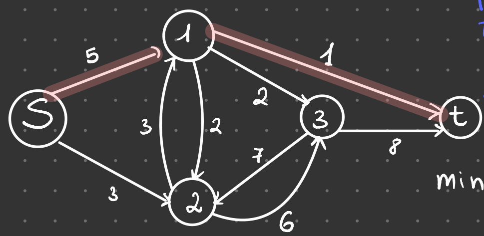
2.2.1 Temi d'esame 2013

Esercizio 2.5. Si determini il flusso massimo tra s e t nel seguente grafo, utilizzando l'Algoritmo di Edmonds e Karp.



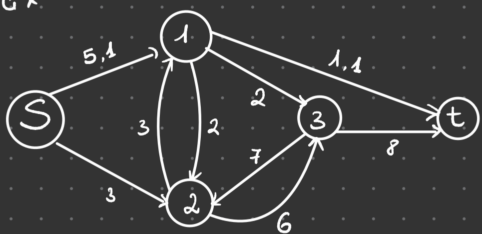
TROVARE IL FLUSSO MASSIMO tra s e t

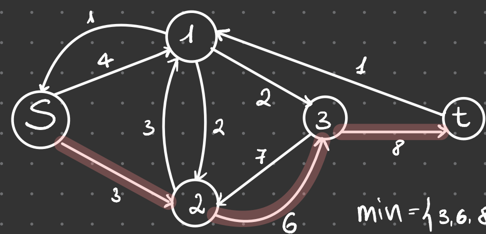
ALGORITMO E-K



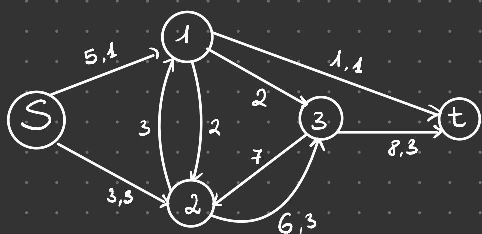
$\min = \{5, 1\} = 1$

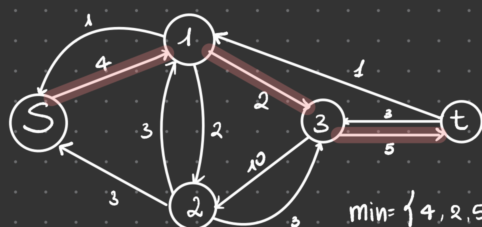
Gx



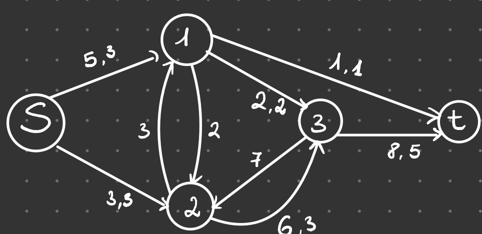


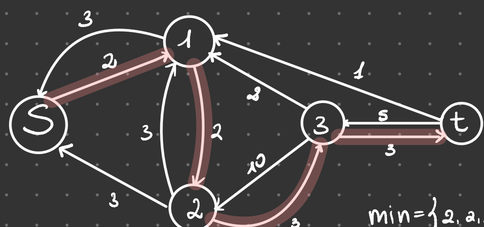
$\min = \{3, 6, 8\} = 3$



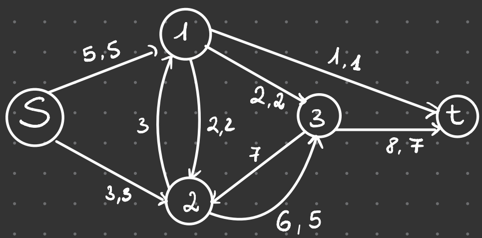


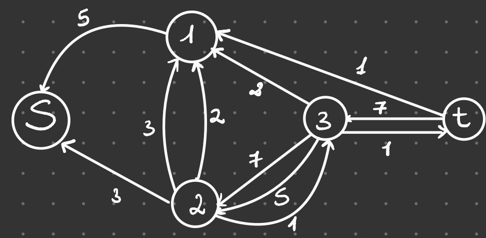
$\min = \{4, 2, 5\} = 2$





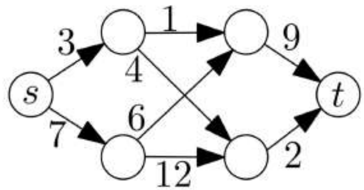
$\min = \{2, 2, 3, 3\} = 2$

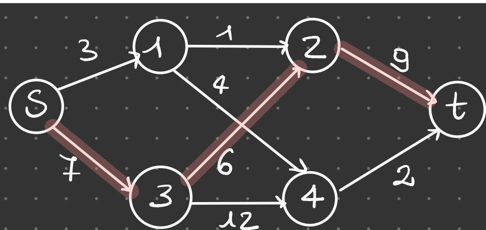




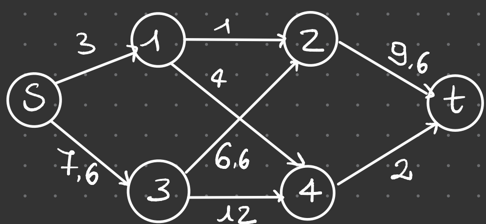
FLUSSO MASSIMO → perchè non esistono più cammino aumentante che va da s a t.

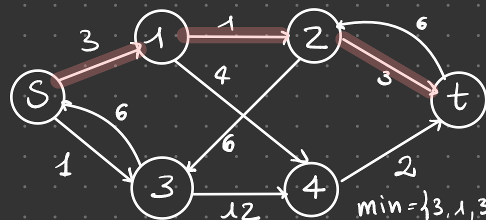
Esercizio 2.6. Si determini il flusso massimo tra s e t nel seguente grafo, utilizzando l'Algoritmo di Edmonds e Karp.



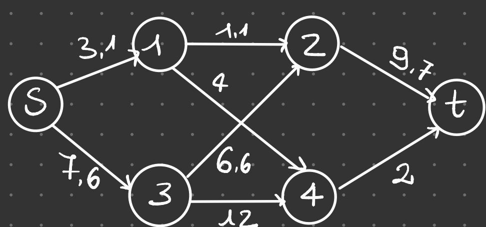


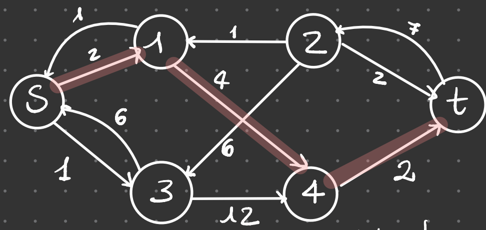
min = {7, 6, 9} = 6



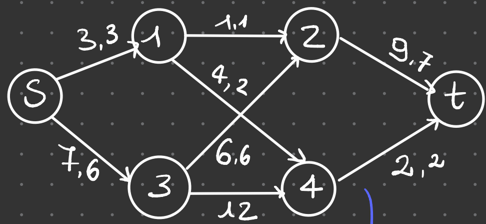


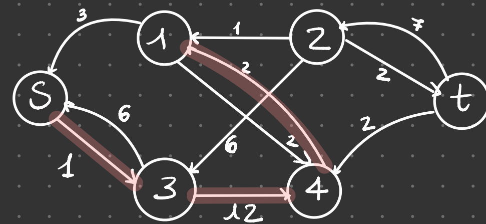
min={3,1,3}





min={2,4,2}
=2

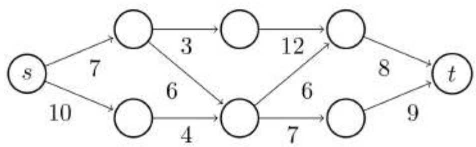


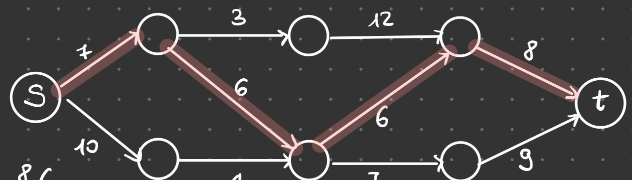


FLUSSO massimo,
non esistono altri cammini aumentanti da s a t.

2.2.2 Temi d'esame 2014

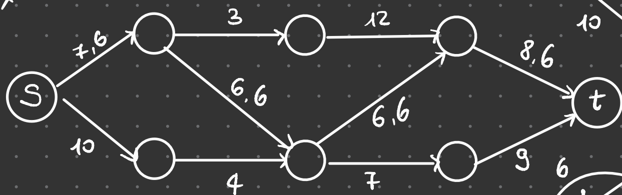
Esercizio 2.8. Si risolva, tramite l'algoritmo di Edmonds e Karp il seguente problema di flusso massimo.

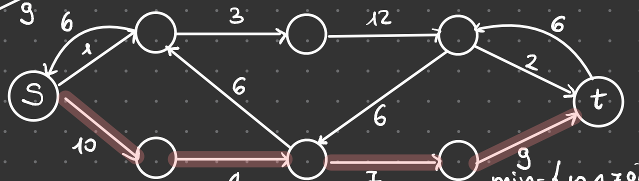




min = {7, 6, 6, 8} = 6

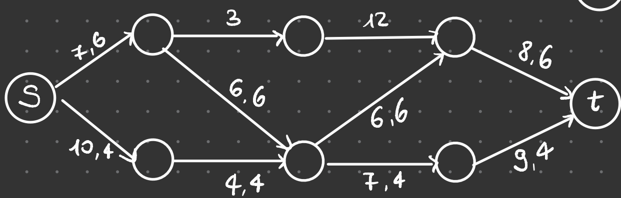
G_x

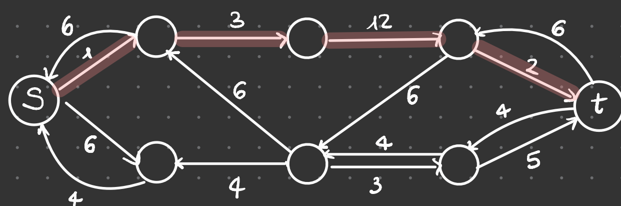




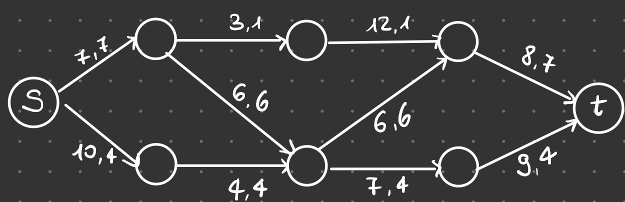
min = {10, 4, 7, 9} = 4

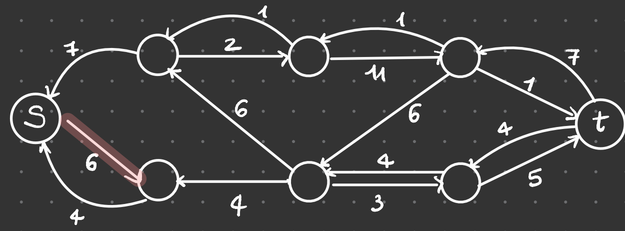
G_x





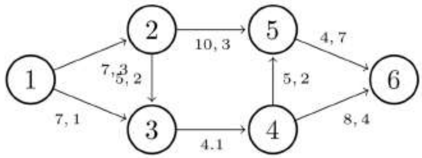
min = {1, 3, 12, 2} = 1



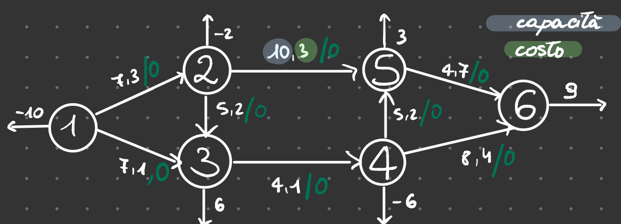


FLUSSO massimo,
non esistono altri cammini aumentanti da s a t

Esercizio 2.9. Si risolva, tramite l'algoritmo dei cammini minimi ~~aumentanti~~ successivi, il seguente problema MCF.



Il vettore b è $(-10, -2, 6, -6, 3, 9)$. Le etichette sugli archi indicano al solito la capacità (il primo numero) e il costo (il secondo numero).

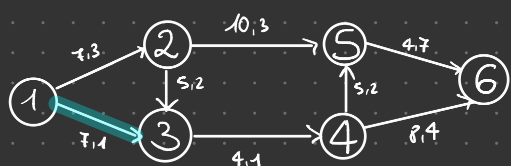


capacità
costo

nodo 1 $10-0=10>0$
nodo 2 $2-0=2>0$
nodo 3 $0-6=-6<0$
nodo 4 $6-0=6>0$
nodo 5 $0-3=-3<0$
nodo 6 $0-9=-9<0$

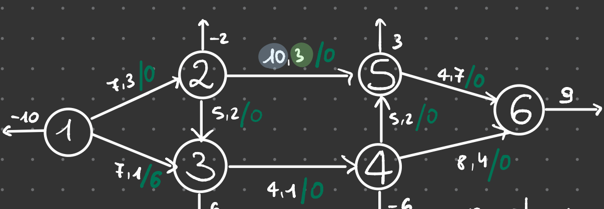
+	−
1	3
2	5
4	6

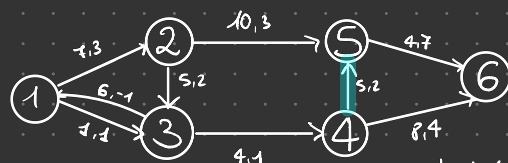
1° cammino di costo minimo che va dal nodo pos al nodo negativo.



$1-3=1$

$\min=\{10, 7, 6\}=6$

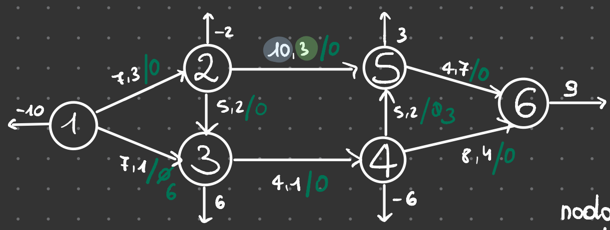


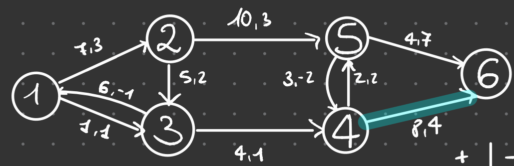


nodo 1 $10-6=4$
nodo 2 $2-0=2$
nodo 3 $6-6=0$
nodo 4 $6-0=+6$
nodo 5 $0-3=-3$
nodo 6 $0-9=-9$

+	−
1	5
2	6
4	

$\min = \{6, 5, 3\} = 3$





nodo 1 $10-6 = 4 > 0$

nodo 2 $2-0 = 2 > 0$

nodo 3 $6-6 = 0$

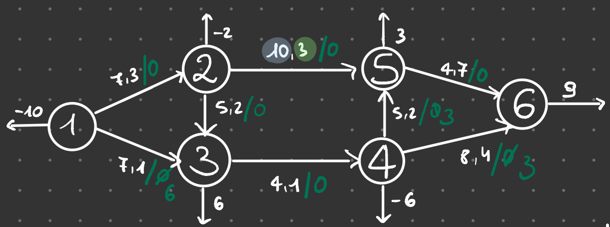
nodo 4 $6-3 = 3 > 0$

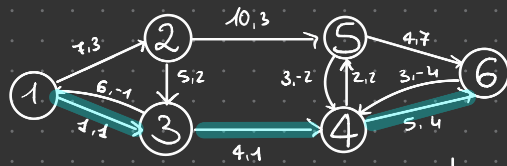
nodo 5 $3-3 = 0$

nodo 6 $0-9 = -9 < 0$

+	-
1	6
2	
4	

$\min = \{3, 8, 9\} = 3$





nodo 1 $10-6 = 4 > 0$

nodo 2 $2-0 = 2 > 0$

nodo 3 $6-6 = 0$

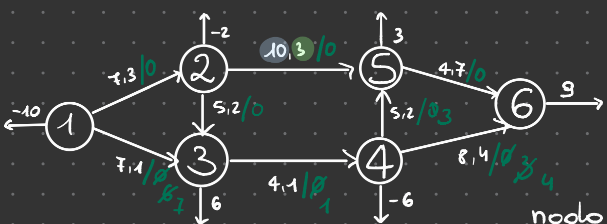
nodo 4 $6-3-3 = 0$

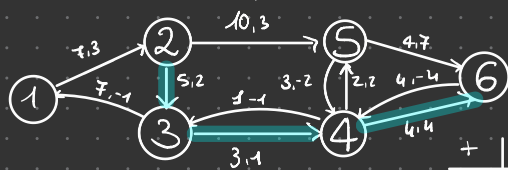
nodo 5 $3-3 = 0$

nodo 6 $3-9 = -6 < 0$

+	-
1	6
2	

$\min = \{4, 1, 4, 5, 6\} = 1$





nodo 1 $10-7 = 3 > 0$

nodo 2 $2-0 = 2 > 0$

nodo 3 $7-6-1 = 0$

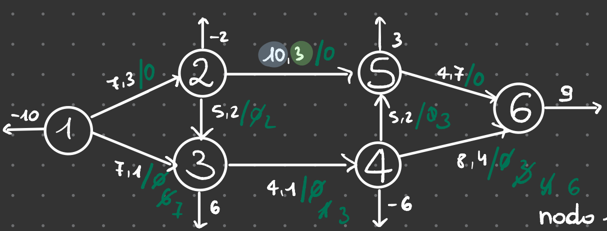
nodo 4 $6+1-3-4 = 0$

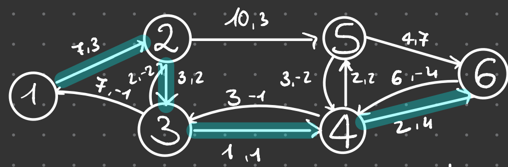
nodo 5 $3-3 = 0$

nodo 6 $4-9 = -5 < 0$

+	-
1	6
2	

$\min = \{2, 5, 3, 4, 5\} = 2$





nodo 1 $10-7 = 3 > 0$

nodo 2 $2-2 = 0$

nodo 3 $7+2-6-3 = 0$

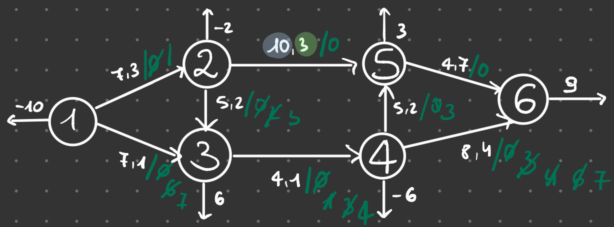
nodo 4 $6+3-3-6 = 0$

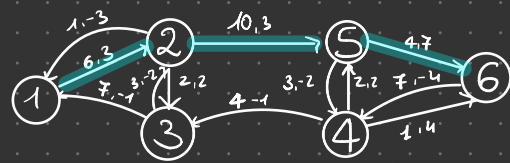
nodo 5 $3-3 = 0$

nodo 6 $6-9 = -3 < 0$

+	-
1	6

$\min = \{3, 7, 3, 1, 2, 3\} = 1$



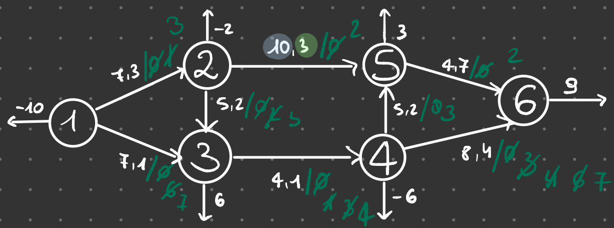


+	-
1	6

nodo 1 = 10-7-1 = 2 > 0

nodo 6 = 7-9 = -2 < 0

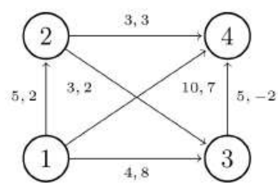
min = {2, 6, 10, 4, 2} = 2



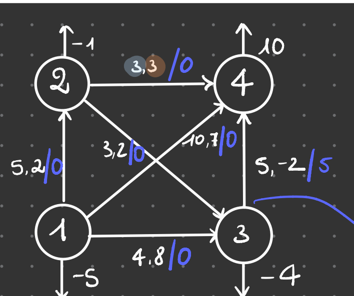
nodo 1 = 10-7-3 = 0

nodo 6 = 7+2-9 = 0

Esercizio 2.10. Si risolva, tramite l'algoritmo dei cammini minimi successivi, il seguente problema MCF.

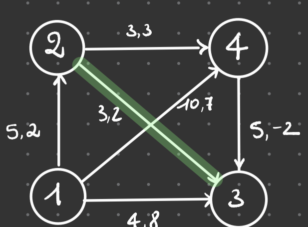


Il vettore b è $(-5, -1, -4, 10)$. Le etichette sugli archi indicano al solito la capacità (il primo numero) e il costo (il secondo numero).



costo > 0 → flusso = 0

costo < 0 → flusso = capacità



Sbilanciamenti

nodo 1 = 5 - 0 = 1 > 0

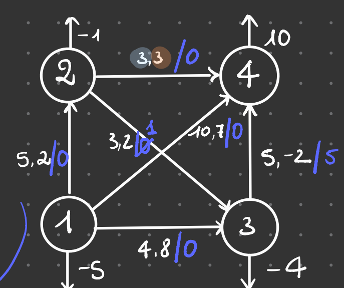
nodo 2 = 1 - 0 = 1 > 0

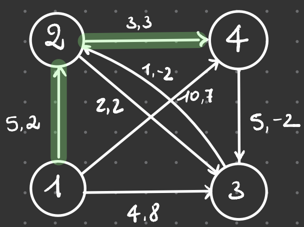
nodo 3 = 4 - 5 = -1 < 0

nodo 4 = 5 - 10 = -5 < 0

min = {1, 3, 1} = 1

+	-
1	3
2	4





Sbilanciamenti

nodo 1 = 5 - 0 = 5 > 0

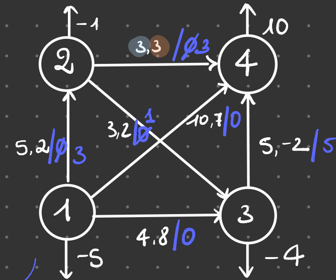
nodo 2 = 1 - 1 = 0

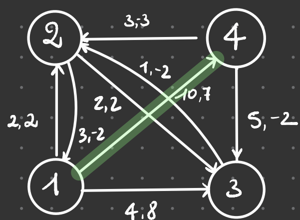
nodo 3 = 4 - 5 + 1 = 0

nodo 4 = 5 - 10 = -5 < 0

+	−
1	4

min = {5, 5, 3, 5} = 3





Sbilanciamenti

nodo 1 = 5 - 3 = 2

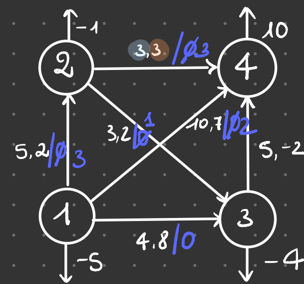
nodo 2 = 1 + 3 - 3 - 1 = 0

nodo 3 = 0

nodo 4 = 3 + 5 - 10 = -2

+	−
1	4

min = {2, 7, 2}



Sbilanciamenti

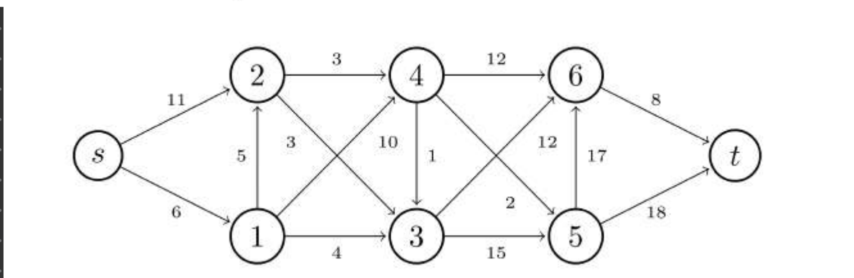
nodo 1 = 5 - 3 - 2 = 0

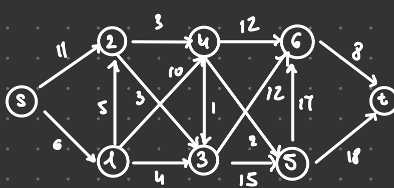
nodo 2 = 1 + 3 - 3 - 1 = 0

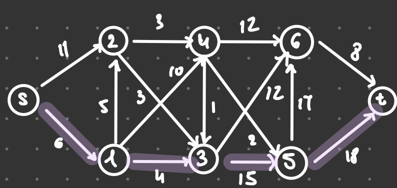
nodo 3 = 4 + 1 - 5 = 0

nodo 4 = 3 + 5 - 10 + 2 = 0

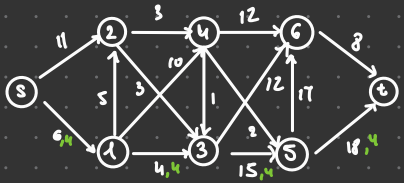
Esercizio 2.11. Si determini il flusso massimo tra s e t nel seguente grafo,utilizzando l'Algoritmo di Edmonds e Karp.

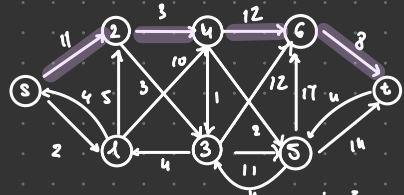




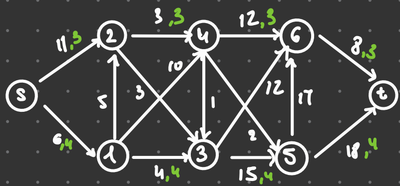


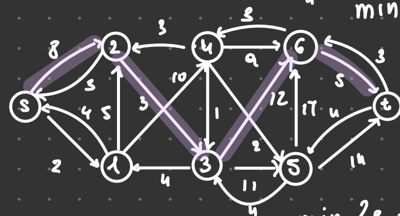
min = {6, 4, 15, 18} = 4



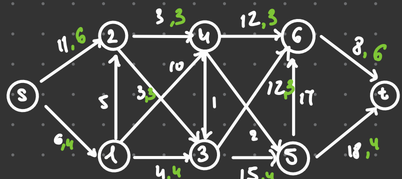


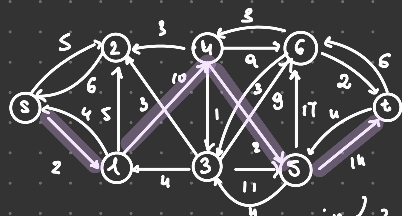
$\min\{11, 3, 12, 8\} = 3$



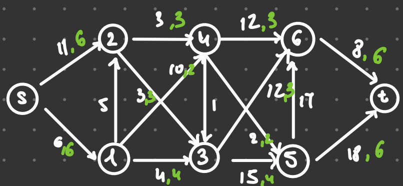


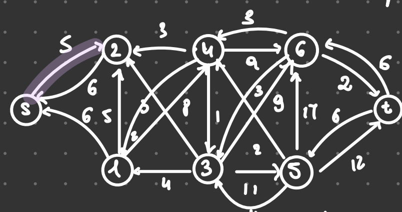
$\min\{8, 3, 12, 5\} = 3$





$\min\{2, 10, 2, 14\} = 2$



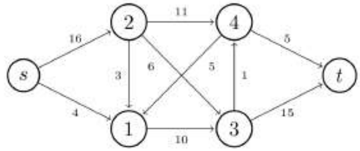


min

$\nexists$ cammini aumentanti

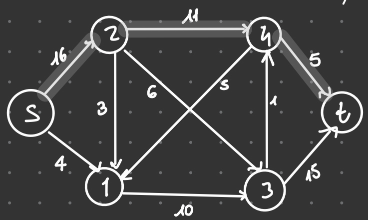
2.2.3 Temi d'esame 2015

Esercizio 2.13. Si risolva, tramite l'algoritmo di Edmonds e Karp, il seguente problema di flusso massimo.

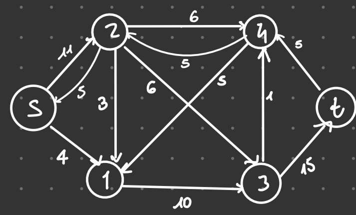


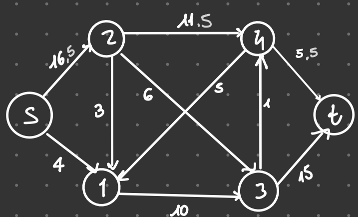
Si dia inoltre un taglio di capacità minima per la rete di cui sopra.

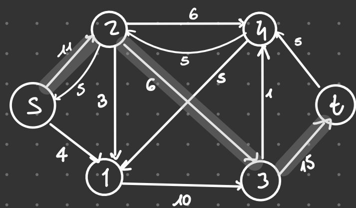
Ed. Karp → cammino minimo (più veloce)



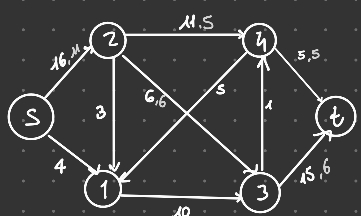
$\min\{16, 11, 5\} = 5$

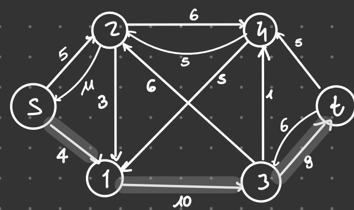




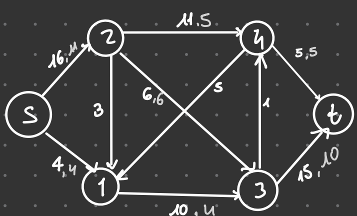


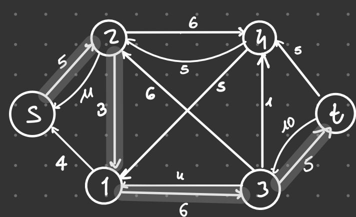
$\min\{11, 6, 15\} = 6$



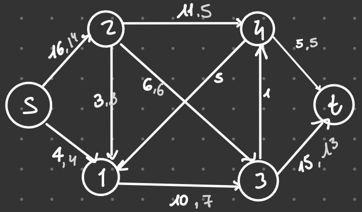


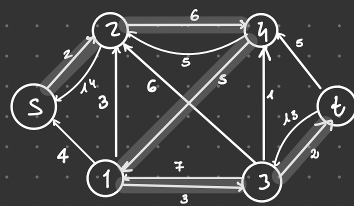
$\min\{4, 10, 9\} = 4$



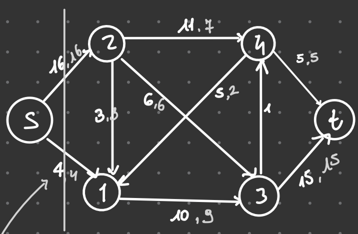


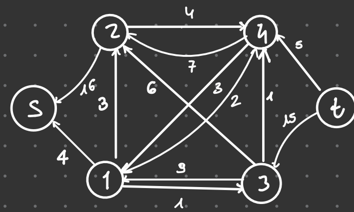
$\min\{5, 3, 6, 5\}$
$= 3$





$\min\{2,6,5,3\} = 2$

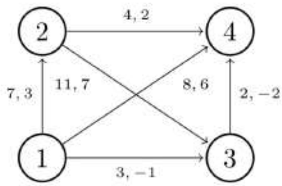




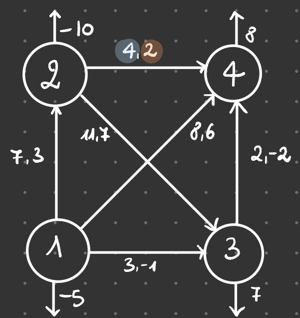
∄ cammini minimi che collegano s a t

Per trovare il taglio usiamo il teorema Max Flw - MIN cut

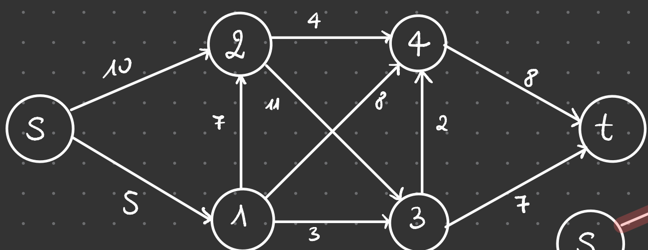
Esercizio 2.15. Si risolva, tramite l'algoritmo basato sull'eliminazione di cicli, il seguente problema MCF.

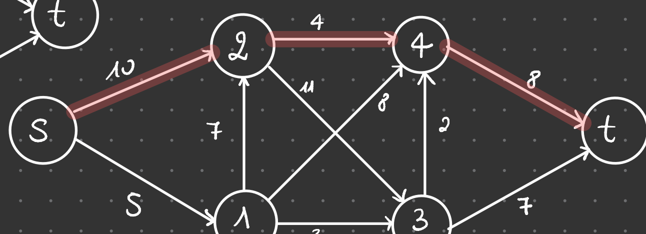


Il vettore b è $(-5, -10, 7, 8)$. Le etichette sugli archi indicano al solito la capacità (il primo numero) e il costo (il secondo numero).



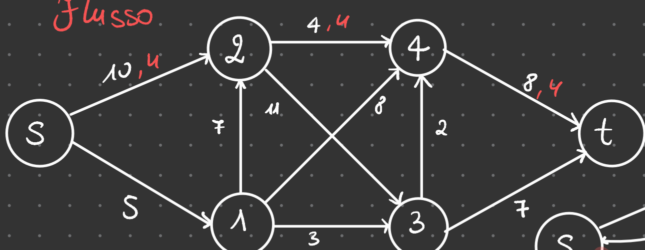
E-K ⟶ flusso massimo

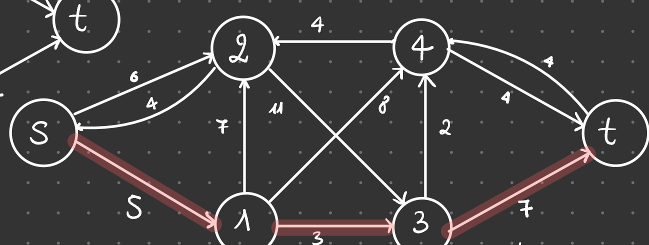




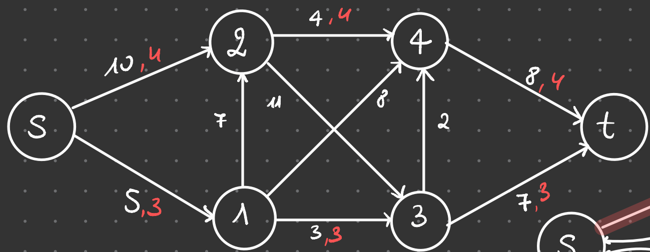
min = {10, 4, 8} = 4

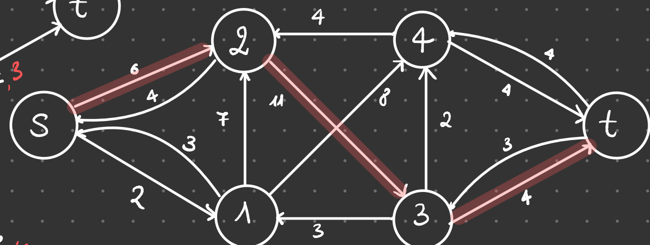
Flusso



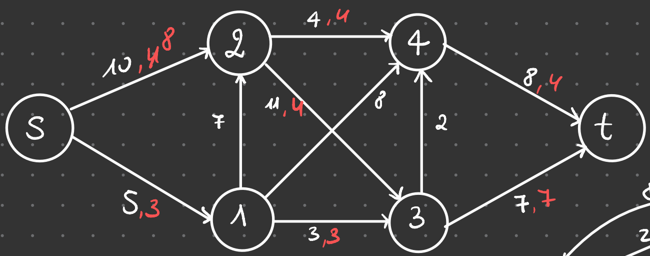


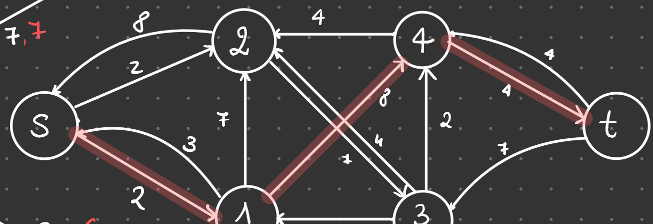
min = {5, 3, 7} = 3



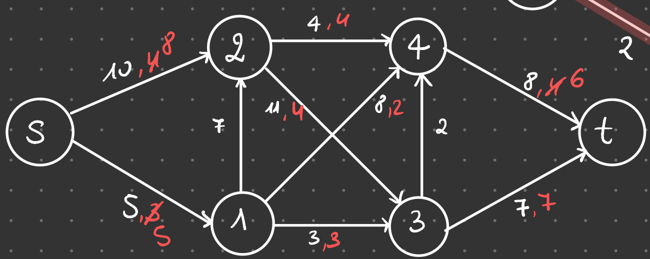


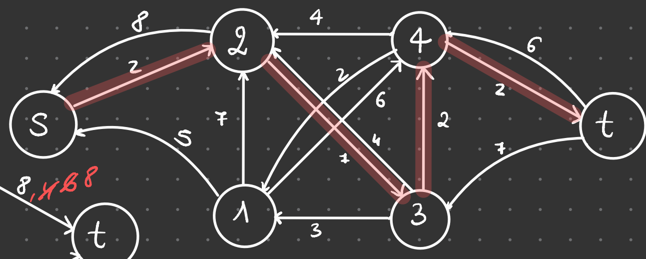
$\min = \{6, 11, 4\} = 4$

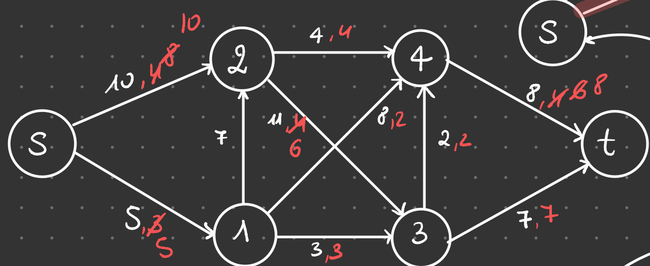




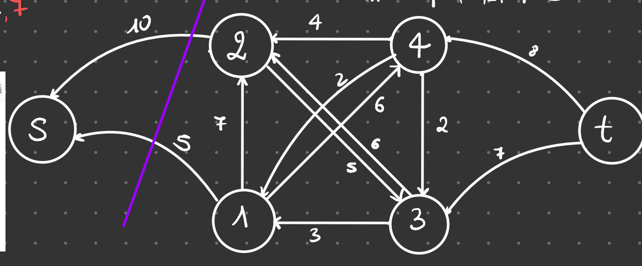
$\min = \{2, 8, 4\} = 2$



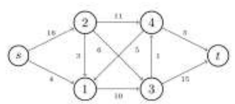




$\min = \{2, 7, 2, 2\} = 2$



Esercizio 2.16. Si risolva, tramite l'algoritmo di Edmonds e Karp, il seguente problema di flusso massimo.



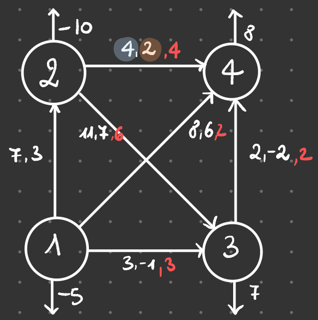
Si dia inoltre un taglio di capacità minima per la rete di cui sopra.

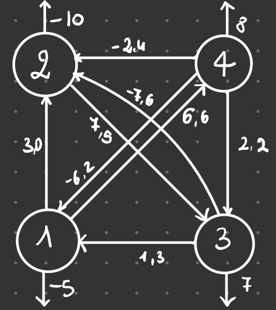
TEOREMA MAX-FLOW MIN CUT

↓

MASSIMO FLUSSO = MINIMA CAPACITÀ DEI TAGLI

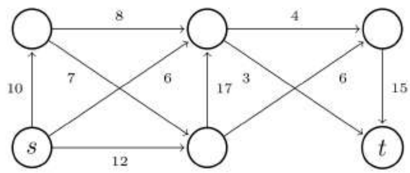
non ∃ più cam. min. aument.





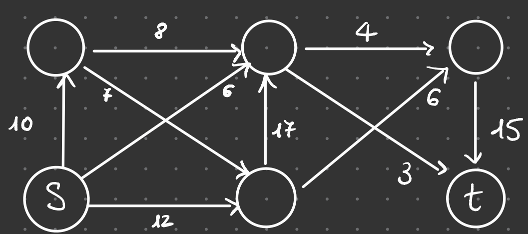
non esistono cicli di costo negativo.

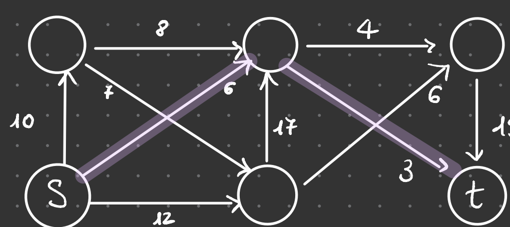
Esercizio 2.17. Si risolva, tramite l'algoritmo di Edmonds e Karp, il seguente problema MF.



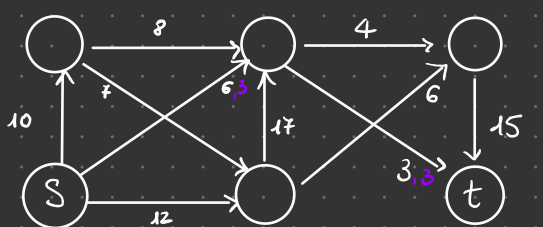
Si dia inoltre un taglio di capacità minima.

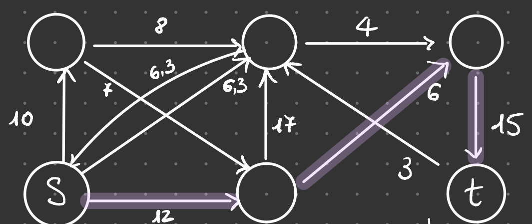
Edmond Karp ⟶ cammino minimo



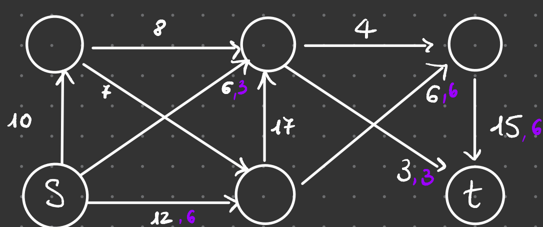


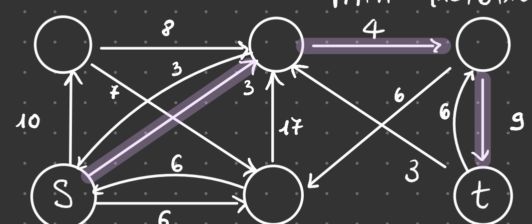
min = {6, 3} = 3



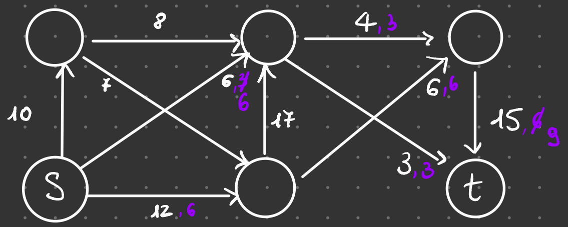


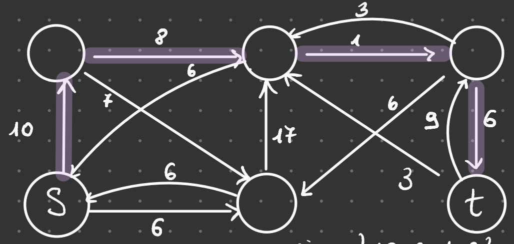
min = {12, 6, 15} = 6



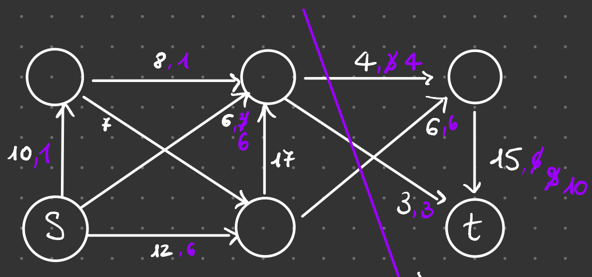


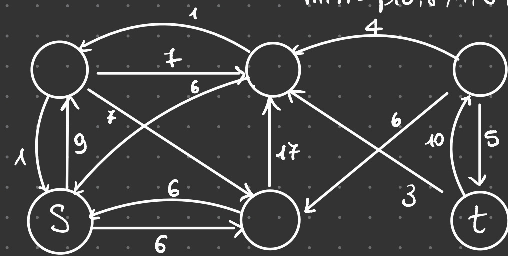
min = {3, 4, 9} = 3





min = {10, 8, 1, 6} = 1



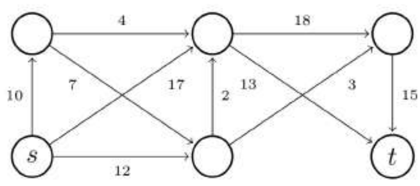


→ flusso massimo → ∄ cammini aumentanti di lunghezza minima che va da s a t.

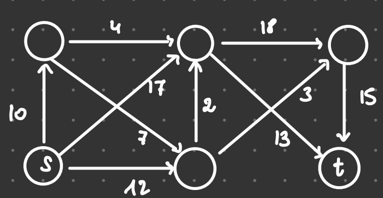
Taglio → usiamo il teorema Max-Flow Min-Cut.

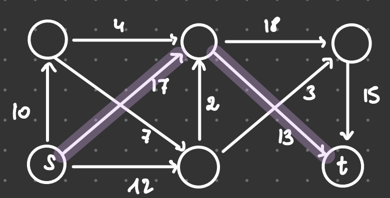
flusso massimo = capacità minima del taglio.

Esercizio 2.18. Si risolva, tramite l'algoritmo di Edmonds e Karp, il seguente problema MF.

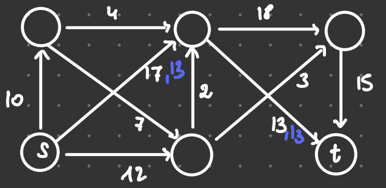


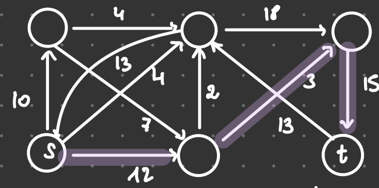
Si dia inoltre un taglio di capacità minima.



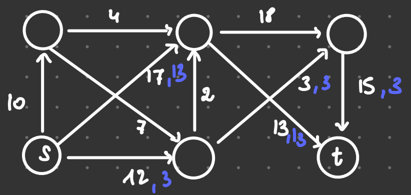


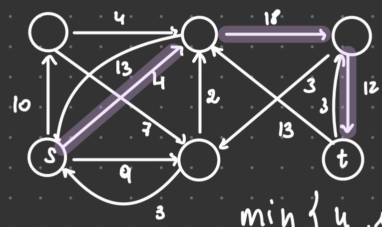
min {17, 13} = 13



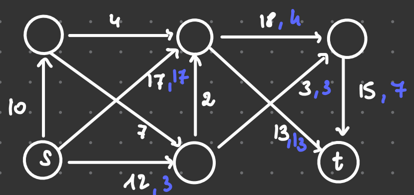


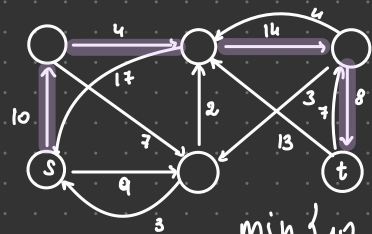
min{12,3,15}=3



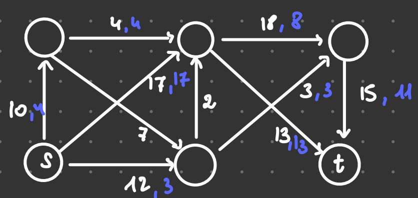


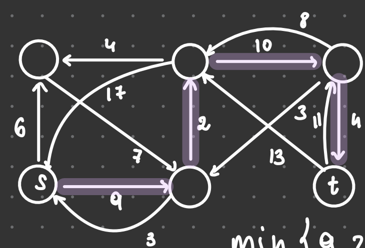
min{4,18,12}=4



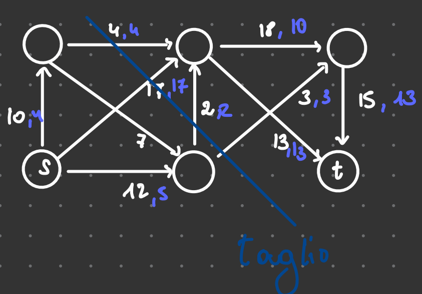


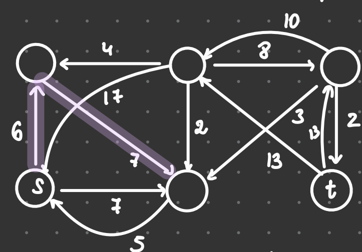
min{10,4,14,8}=4





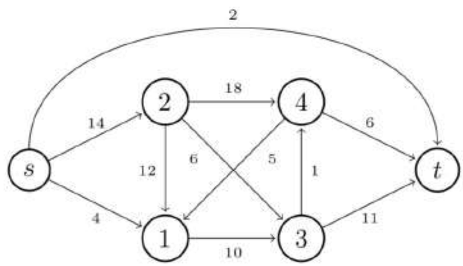
min{9,2,10,4}=2



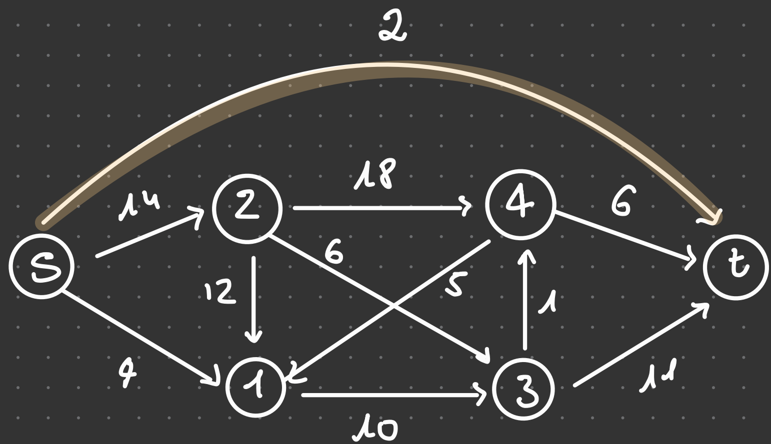


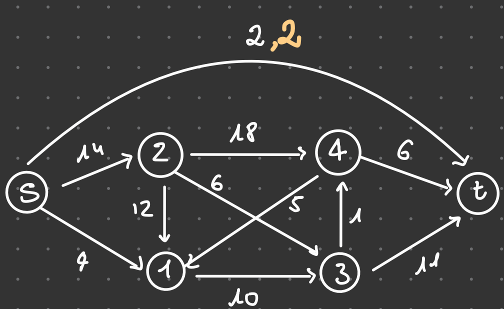
flusso massimo = capacità minima

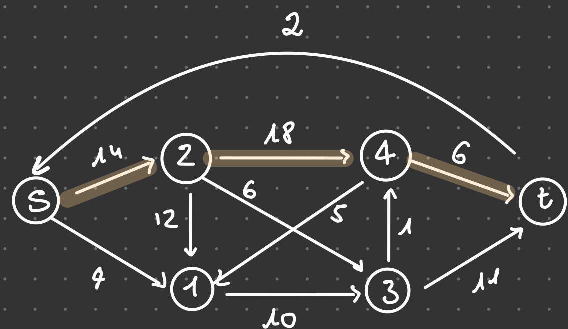
Esercizio 2.19. Si risolva, tramite l'algoritmo di Edmonds e Karp, il seguente problema di flusso massimo.



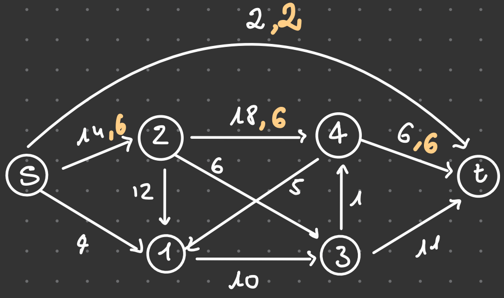
Si dia inoltre un taglio di capacità minima per la rete di cui sopra.

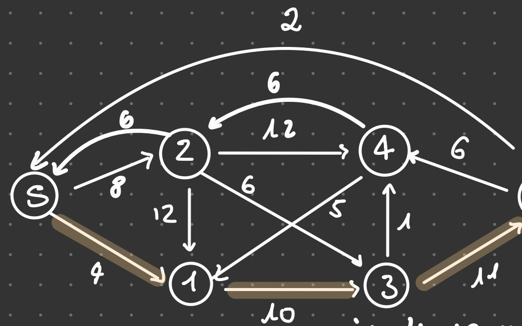




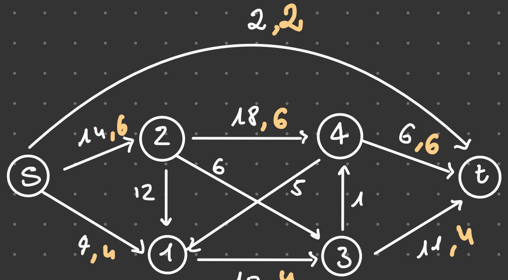


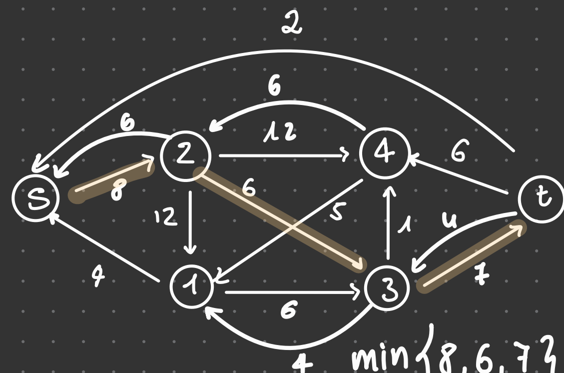
$\min \{14, 18, 6\} = 6$



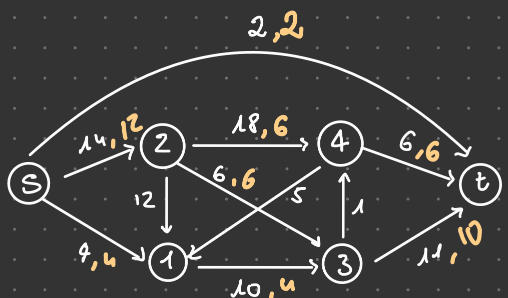


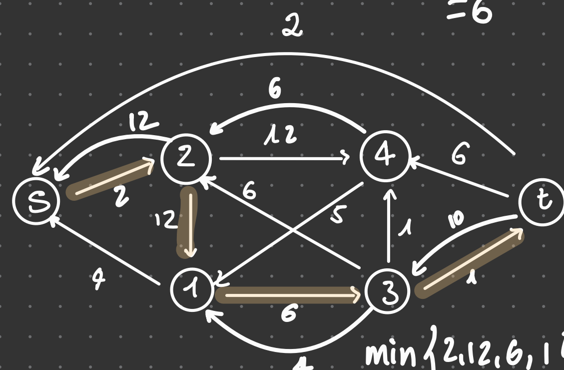
min {4, 10, 11} = 4



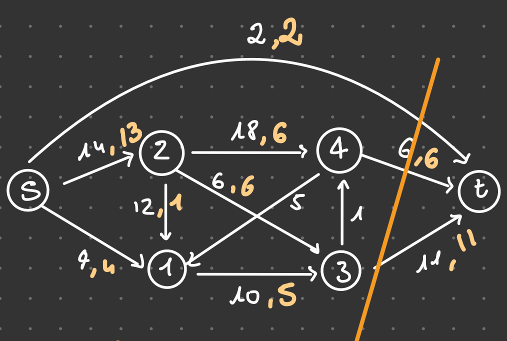


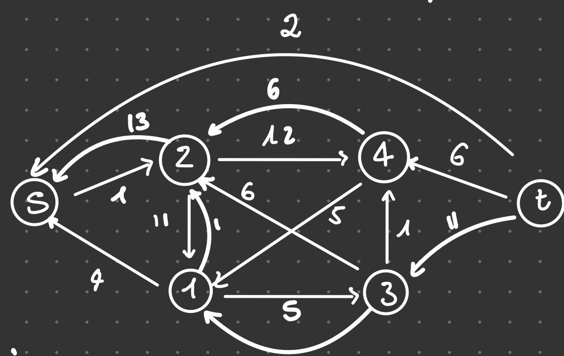
min{8, 6, 7}
=6





min{2,12,6,1}=1





max flow - min cut

massimo flusso = capacità minima