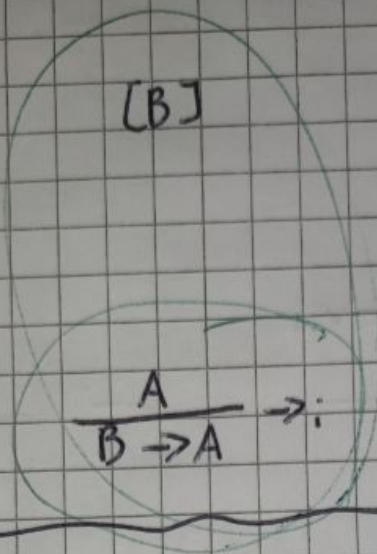


Esercizio 1

$$A \vdash B \rightarrow A$$



$$B \vdash A \rightarrow A$$

[A]

$$\frac{[A]}{A \rightarrow A} \rightarrow_i$$

$$(A \rightarrow A) \vdash (A \rightarrow A)$$

1)

$$A \rightarrow A$$

[A]

2)

$$\frac{[A]}{A \rightarrow A} \rightarrow_i$$

[A]

3)

$$\frac{A \rightarrow A \quad [A]}{A} \rightarrow_e$$
$$\frac{A}{A \rightarrow A} \rightarrow_i$$

AFAVA

1)

$$\frac{A}{AVA} \text{ viz}$$

2)

$$\frac{A}{AVA} \text{ viz}$$

Esercizio 2

- $V \rightarrow M, M \vdash V$

impossibile

V

- $B \rightarrow D, D \rightarrow M \vdash B \rightarrow M$
[B]¹

$$\frac{\frac{D \rightarrow M}{B \rightarrow M} \rightarrow i1}{\frac{B \rightarrow D \quad [B]}{D} \rightarrow e} \rightarrow e$$

- $R \rightarrow L, L \rightarrow M \vdash R \rightarrow M$
[R]¹

$$\frac{\frac{L \rightarrow M}{R \rightarrow M} \rightarrow i1}{\frac{R \rightarrow L \quad [R]}{L} \rightarrow e} \rightarrow e$$

• $C \rightarrow B, P \rightarrow B \vdash C \vee P \rightarrow B$

$[C \vee P]^1$
 $[C]^2$ $[P]^2$

$\frac{C \rightarrow B \quad [C]}{B}$	$\frac{P \rightarrow B \quad [P]}{B}$
$[C \vee P]$	B
<hr/>	
B	
<hr/>	
$C \vee P \rightarrow B \rightarrow 1,1$	

Esercizio 3

$$\vdash (A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C$$

$$[A \rightarrow B \rightarrow C]^1$$

$$[A \rightarrow B]^2$$

$$[A]^3$$

$$\begin{array}{c} \frac{\frac{\frac{[A \rightarrow B \rightarrow C] \quad [A]}{B \rightarrow C} \rightarrow e \quad \frac{[A \rightarrow B] \quad [A]}{B} \rightarrow e}{C} \rightarrow i_3}{A \rightarrow C} \rightarrow i_2}{(A \rightarrow B) \rightarrow A \rightarrow C} \rightarrow i_1}{(A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C} \rightarrow i_1 \end{array}$$

~~$$\begin{array}{c} \frac{\frac{\frac{[A \rightarrow B \rightarrow C] \quad [A \rightarrow B]}{C} \rightarrow e}{A \rightarrow C} \rightarrow i_1}{(A \rightarrow B) \rightarrow A \rightarrow C} \rightarrow i_1}{(A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C} \rightarrow i_1 \end{array}$$~~

Esercizio 4

$$E, A \wedge B \rightarrow C \vee D \vdash ((C \rightarrow D) \rightarrow D \rightarrow A \rightarrow D) \wedge E$$

$$\begin{array}{l} [C \rightarrow D]^1 \\ [B]^2 \\ [A]^3 \end{array}$$

$$[C]^{4L} \quad [D]^{4R}$$

$$\frac{\frac{A \wedge B \rightarrow C \vee D \quad \frac{[A] \quad [B]}{A \wedge B} \wedge_i}{C \vee D} \rightarrow_e \quad \frac{[C \rightarrow D] \quad [C]}{D} \rightarrow_e}{[D]} \vee_e (L-R)$$

$$\frac{\frac{\frac{D}{A \rightarrow D} \rightarrow_i 3 \quad \frac{A \rightarrow D}{B \rightarrow A \rightarrow D} \rightarrow_i 2}{B \rightarrow A \rightarrow D} \rightarrow_i 1 \quad \frac{((C \rightarrow D) \rightarrow B \rightarrow A \rightarrow D) \wedge E}{(C \rightarrow D) \rightarrow B \rightarrow A \rightarrow D} \wedge_i}{((C \rightarrow D) \rightarrow B \rightarrow A \rightarrow D) \wedge E} \wedge_i$$

Esercizio 5

$$\vdash A, (B \rightarrow C \vee D) \wedge (C \rightarrow (A \rightarrow \perp) \vee D) \vdash (A \rightarrow B) \rightarrow D$$

$$\begin{array}{l} [A \rightarrow B]^1 \\ [B \rightarrow C \vee D]^2 \\ [C \rightarrow (A \rightarrow \perp) \vee D]^2 \end{array}$$

$$[D]^{3R}$$

$$[C]^{3L} \quad [A \rightarrow \perp]^{4L} \quad [D]^{4R}$$

$$\frac{\frac{[A \rightarrow B] \quad A}{A} \rightarrow_e \quad \frac{[B \rightarrow C \vee D] \quad B}{C \vee D} \rightarrow_e \quad \frac{[C \rightarrow (A \rightarrow \perp) \vee D] \quad [C]}{[A \rightarrow \perp] \vee D} \rightarrow_e \quad \frac{[A \rightarrow \perp] \quad A}{\perp} \rightarrow_e \quad \frac{\perp \quad D}{D} \rightarrow_e \quad \frac{[D]}{D} \vee_e}{\frac{((B \rightarrow C \vee D) \wedge (C \rightarrow (A \rightarrow \perp) \vee D)) \quad D}{(A \rightarrow B) \rightarrow D} \rightarrow_i 1} \wedge_e 2$$

Esercizio 6

$A \rightarrow D \vee B, D \rightarrow C \rightarrow \perp, B \wedge C \rightarrow E \vdash A \wedge C \rightarrow E$

$[A \wedge C]^1$
 $[A]^2 [C]^2$
 $[D]^3 \perp$

$[B]^3 R$

