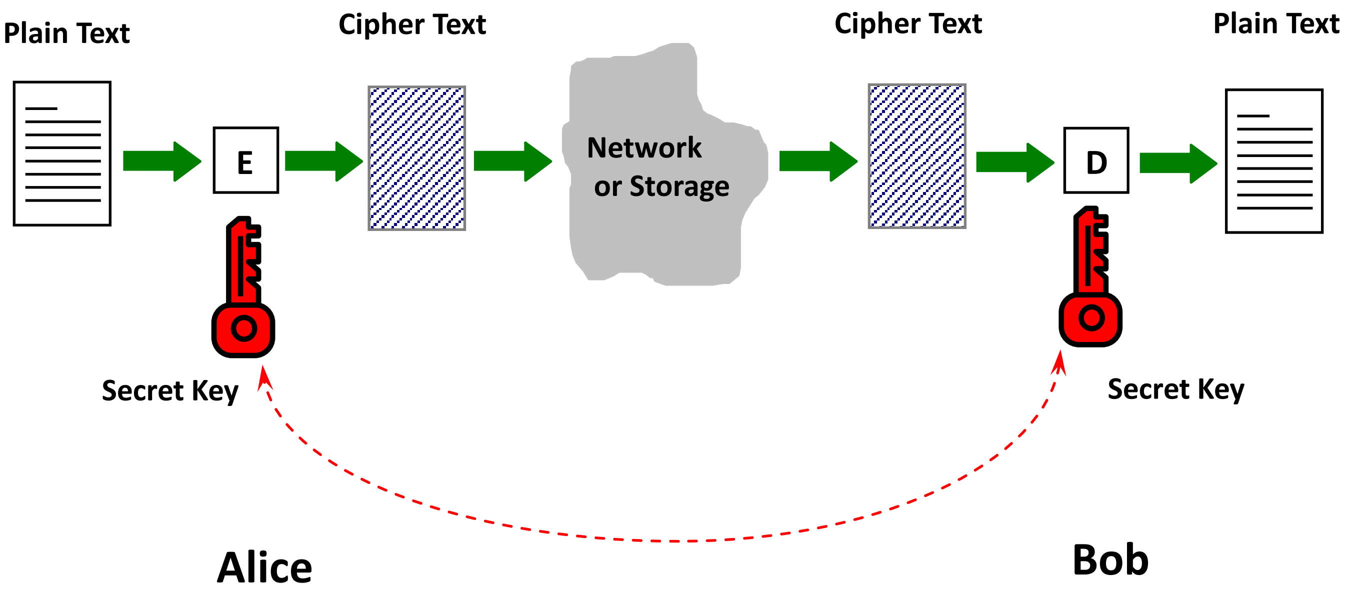
Asymmetric Cryptography

Public key encryption:
definitions and security

Symmetric Cipher



Problems with Symmetric Ciphers

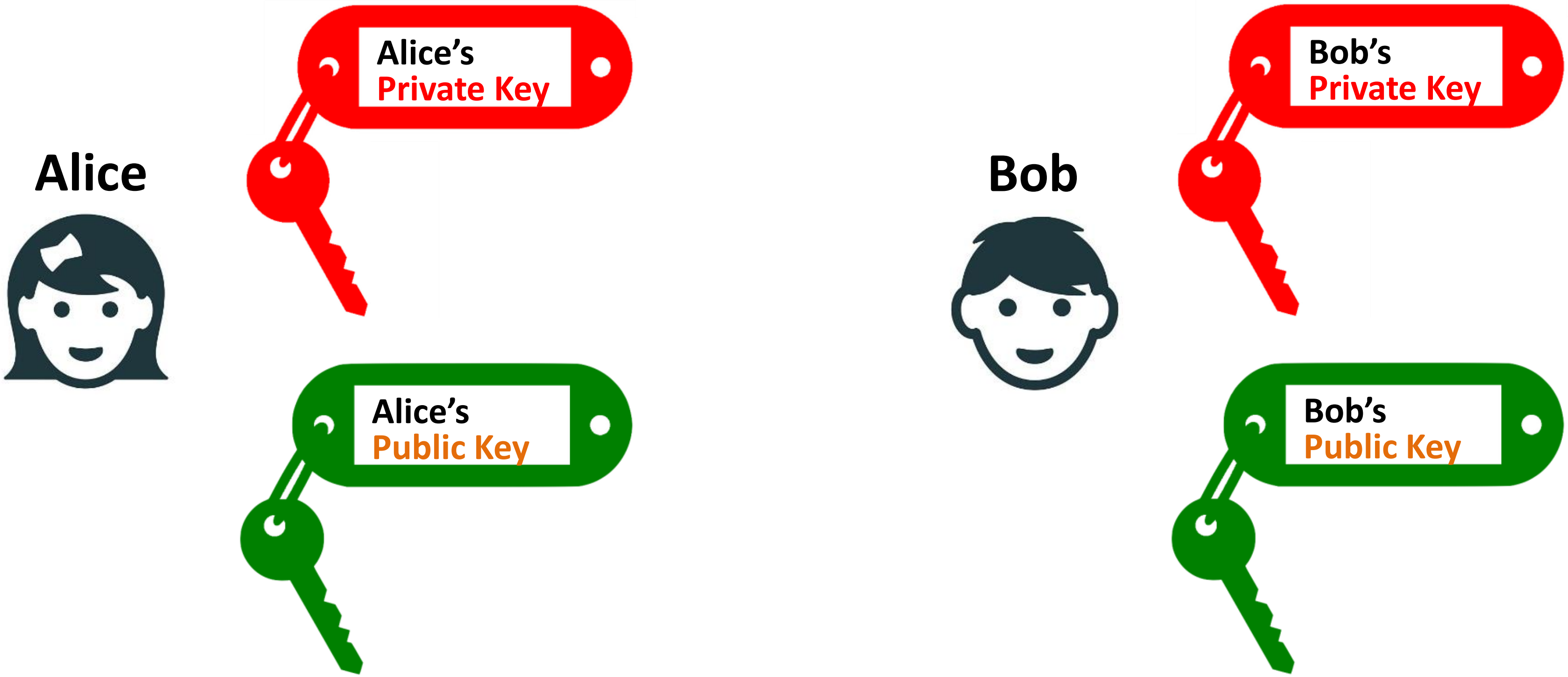
- In order for Alice & Bob to be able to communicate securely using a symmetric cipher, such as AES, they have to have a **shared key** in the first place.
  - What if they have never met before?
- Alice needs to keep 100 different keys if she wishes to communicate with 100 different people

Motivation of Asymmetric Cryptography

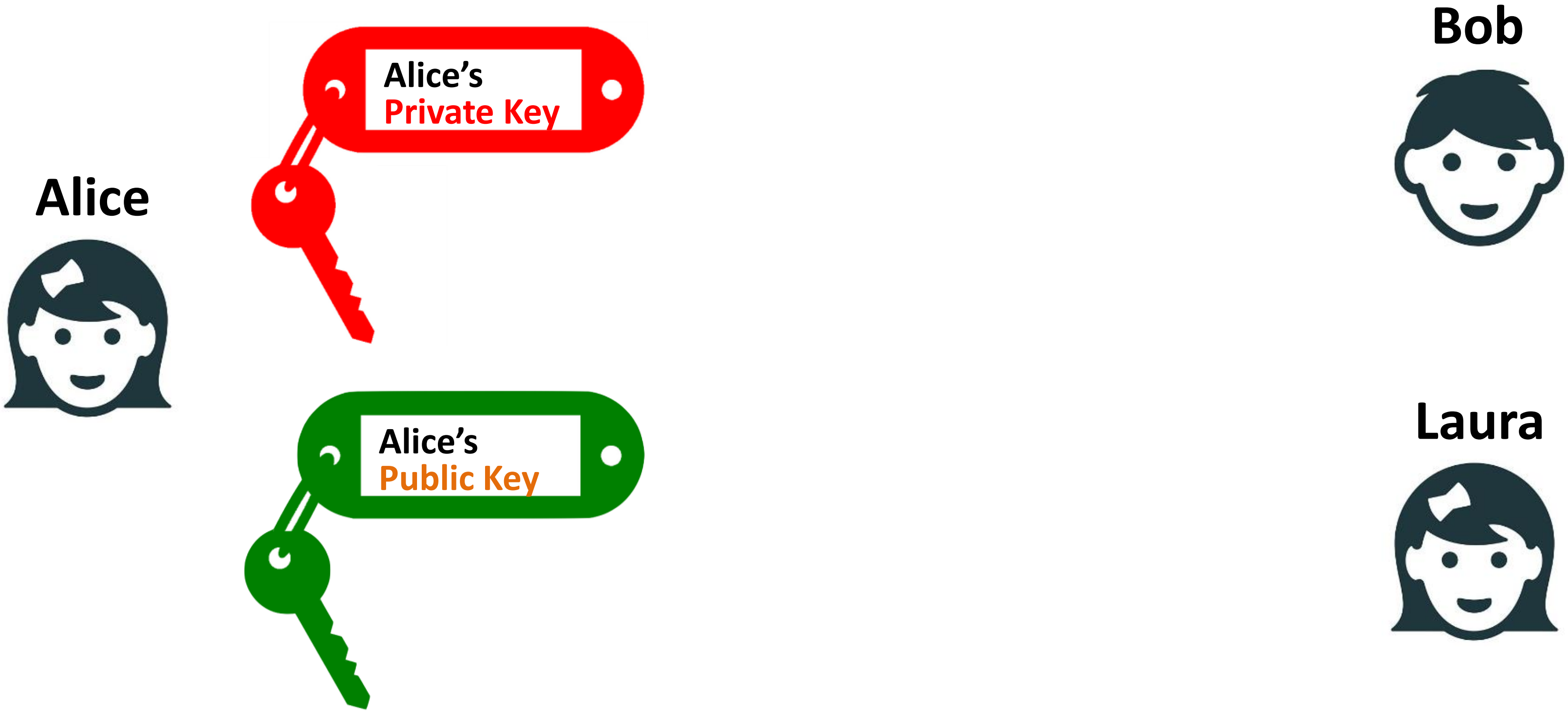
- Is it possible for Alice & Bob, who have no shared secret key, to communicate securely?

- This led to **Asymmetric Cryptography**

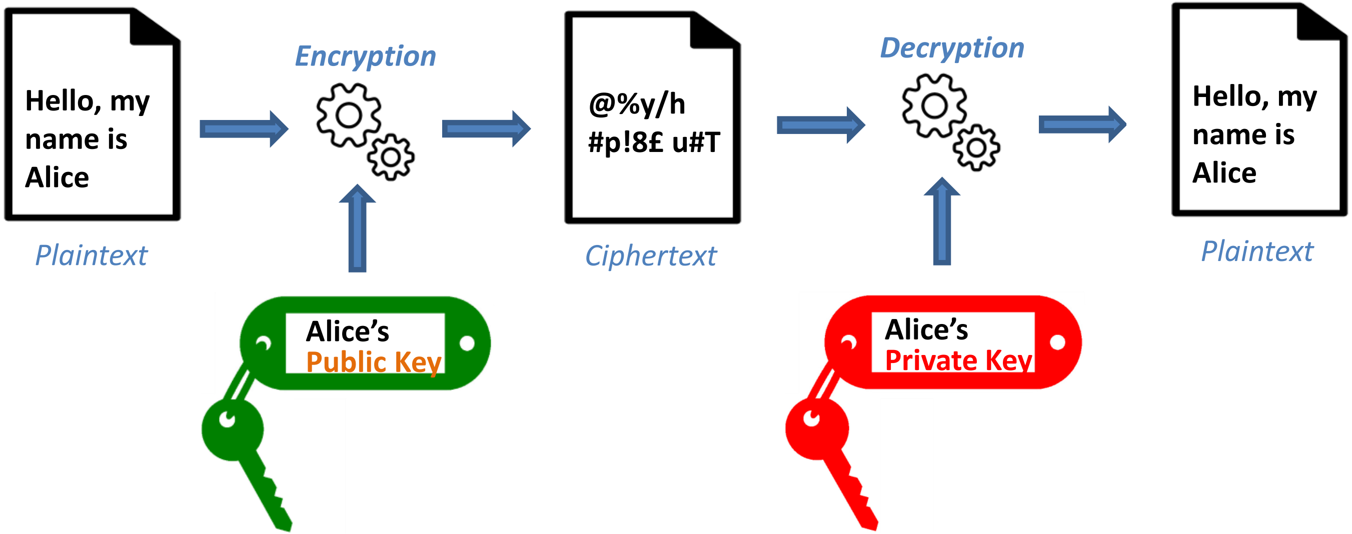
Asymmetric Cryptography



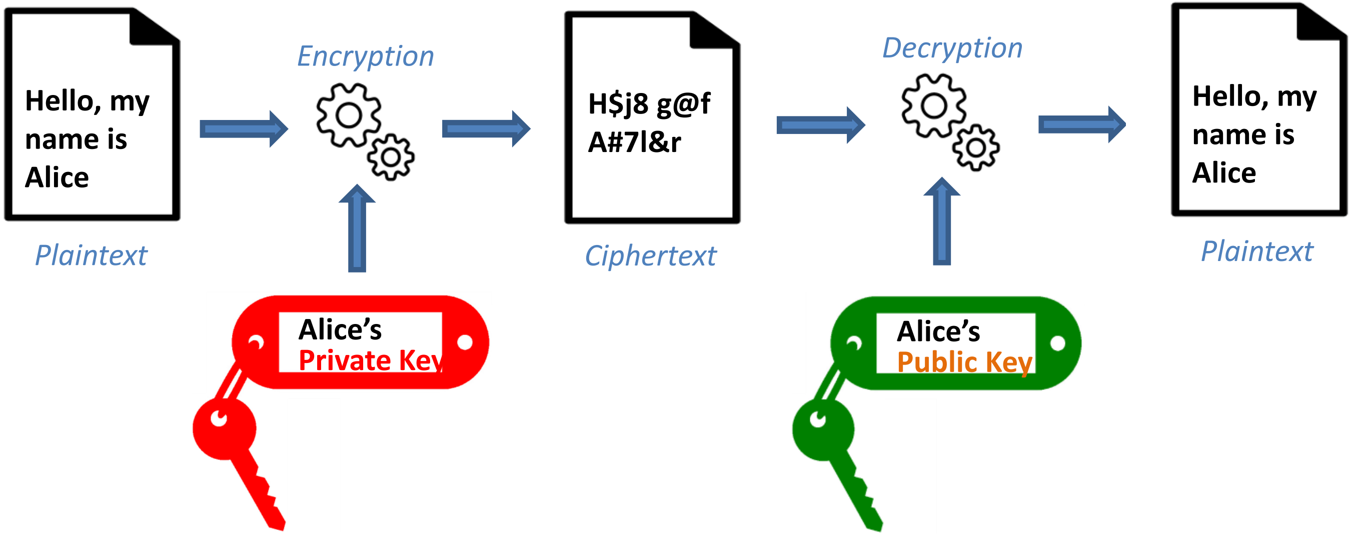
Asymmetric Cryptography



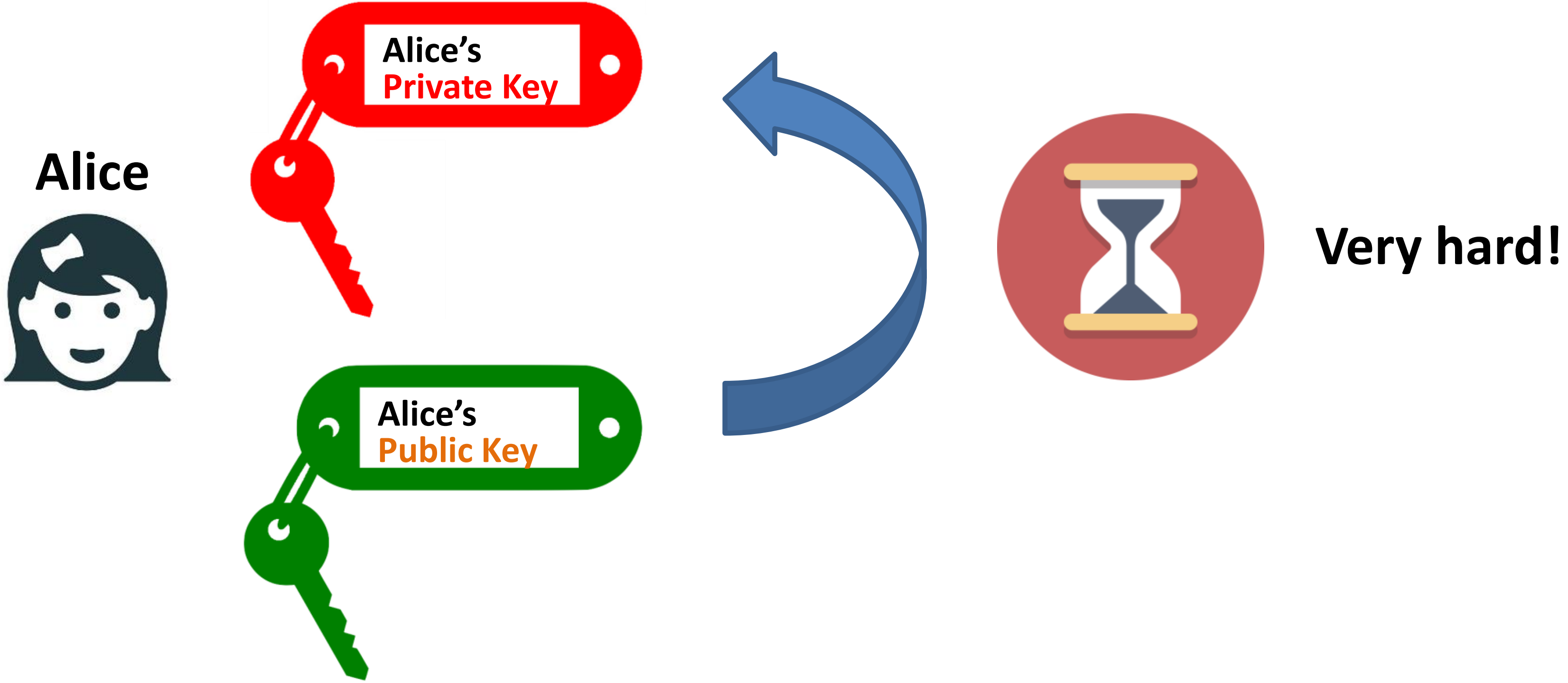
Public and private keys



Public and private keys



Public and private keys



Asymmetric Cryptography

- **Public** key
- **Private** key

- $E(\text{private-key}_{Alice}, m) = c$
- $D(\text{public-key}_{Alice}, c) = m$

- $E(\text{public-key}_{Alice}, m) = c$
- $D(\text{private-key}_{Alice}, c) = m$

Main ideas

- Bob:
  - **publishes**, say in Yellow/White pages, his **public key**, and
  - **keeps** to himself the **matching private key**.

Main ideas (Confidentiality)

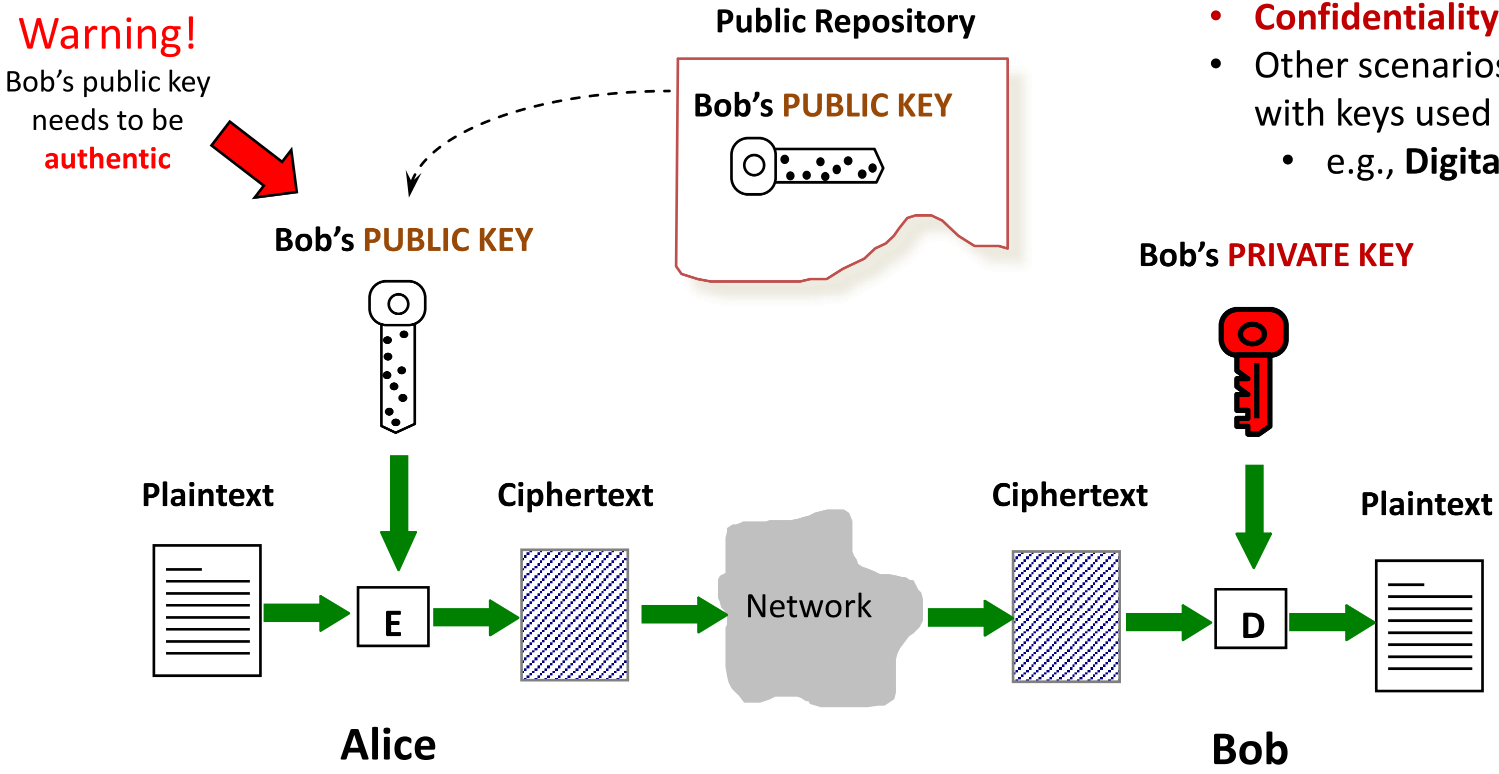
- Alice:
  - Looks up the phone book, and **finds out Bob's public key**
  - **Encrypts a message using Bob's public key** and the encryption algorithm.
  - **Sends the ciphertext** to Bob.

Main ideas (Confidentiality)

- Bob:
  - **Receives the ciphertext** from Alice.
  - **Decrypts** the ciphertext **using his private key**, together with the decryption algorithm

Asymmetric Encryption

- **Confidentiality scenario**
- Other scenarios are possible, with keys used differently…
  - e.g., **Digital signatures**



Main differences with Symmetric Crypto

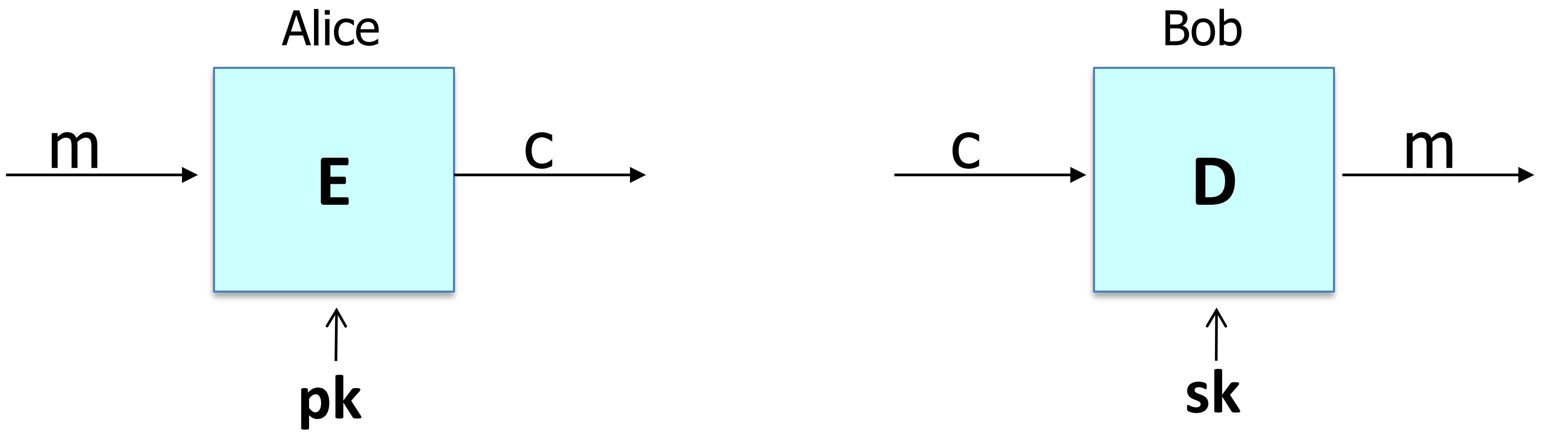
- The public key is different from the private key.
- Infeasible for an attacker to find out the private key from the public key.
- No need for Alice & Bob to distribute a shared secret key beforehand!
- Only one pair of public and private keys is required for each user!

Let’s start seriously

- define what is public key encryption
- what it means for public key encryption to be secure

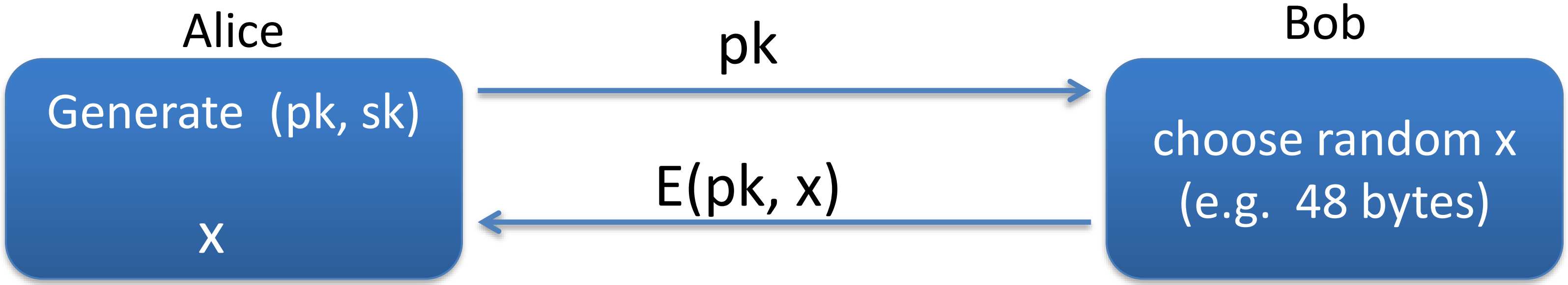
Public key encryption

Bob: generates (PK, SK) and gives PK to Alice



Applications

Session setup (for now, only eavesdropping security)



Non-interactive applications: (e.g. Email)

- Bob sends email to Alice encrypted using pk_{alice}
- Note: Bob needs pk_{alice} (public key management)

Public key encryption

Def: a public-key encryption system is a triple of algs. (G, E, D)

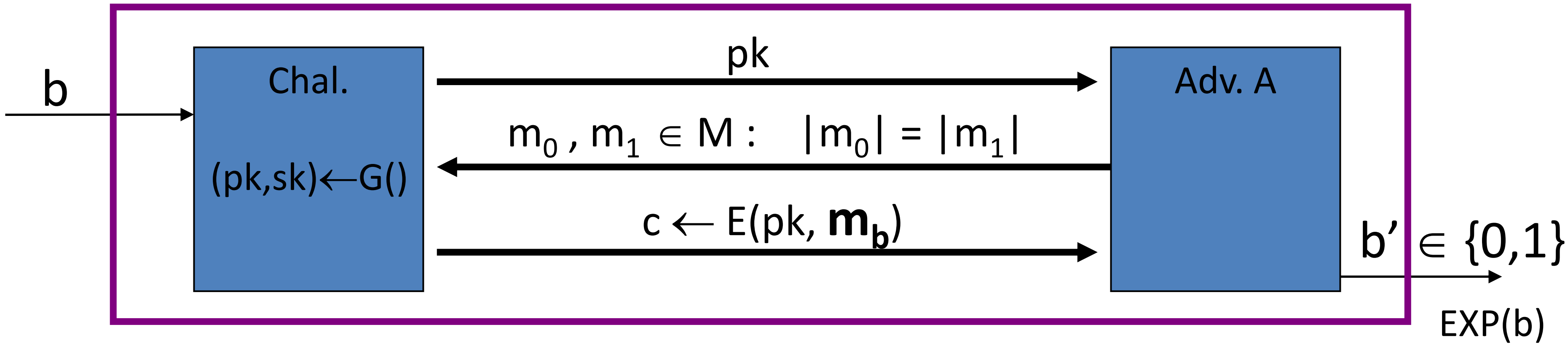
- G(): randomized alg. outputs a key pair (pk, sk)
- E(pk, m): randomized alg. that takes $m \in M$ and outputs $c \in C$
- D(sk,c): det. alg. that takes $c \in C$ and outputs $m \in M$ or $\perp$

Consistency: $\forall$(pk, sk) output by G :

$$\forall m \in M: \quad D(sk,\ E(pk, m)\ ) = m$$

Security: eavesdropping

For b=0,1 define experiments EXP(0) and EXP(1) as:



Def: $\mathbb{E}$ =(G,E,D) is sem. secure (a.k.a IND-CPA) if for all efficient A:

$$\text{Adv}_{SS}[A,\mathbb{E}] = \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right| < \text{negligible}$$

Relation to symmetric cipher security

Recall: for symmetric ciphers we had two security notions:

- One-time security and many-time security (CPA)
- We showed that one-time security $\not\Rightarrow$ many-time security

For public key encryption:

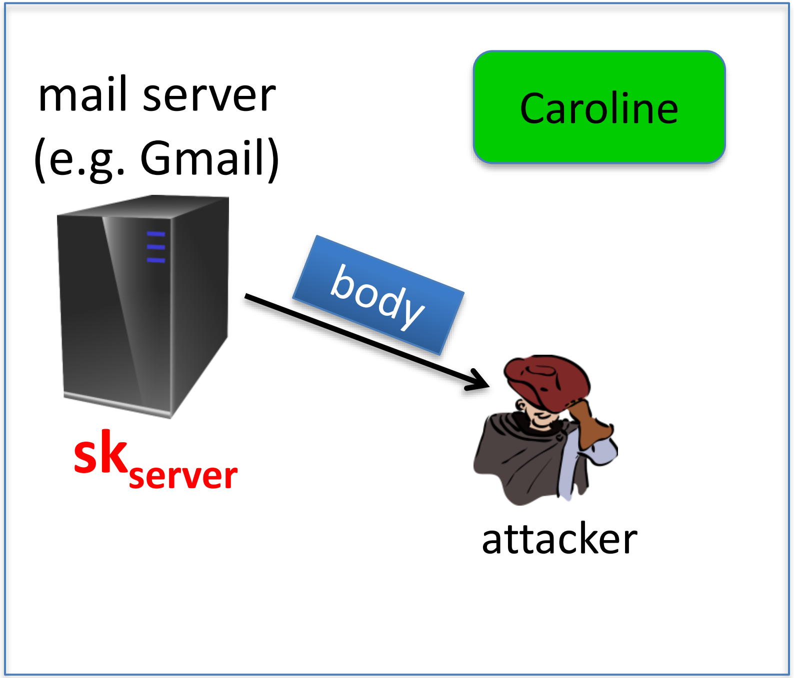
- One-time security $\Rightarrow$ many-time security (CPA)

  (follows from the fact that attacker can encrypt by himself)

- Public key encryption **must** be randomized

Security against active attacks

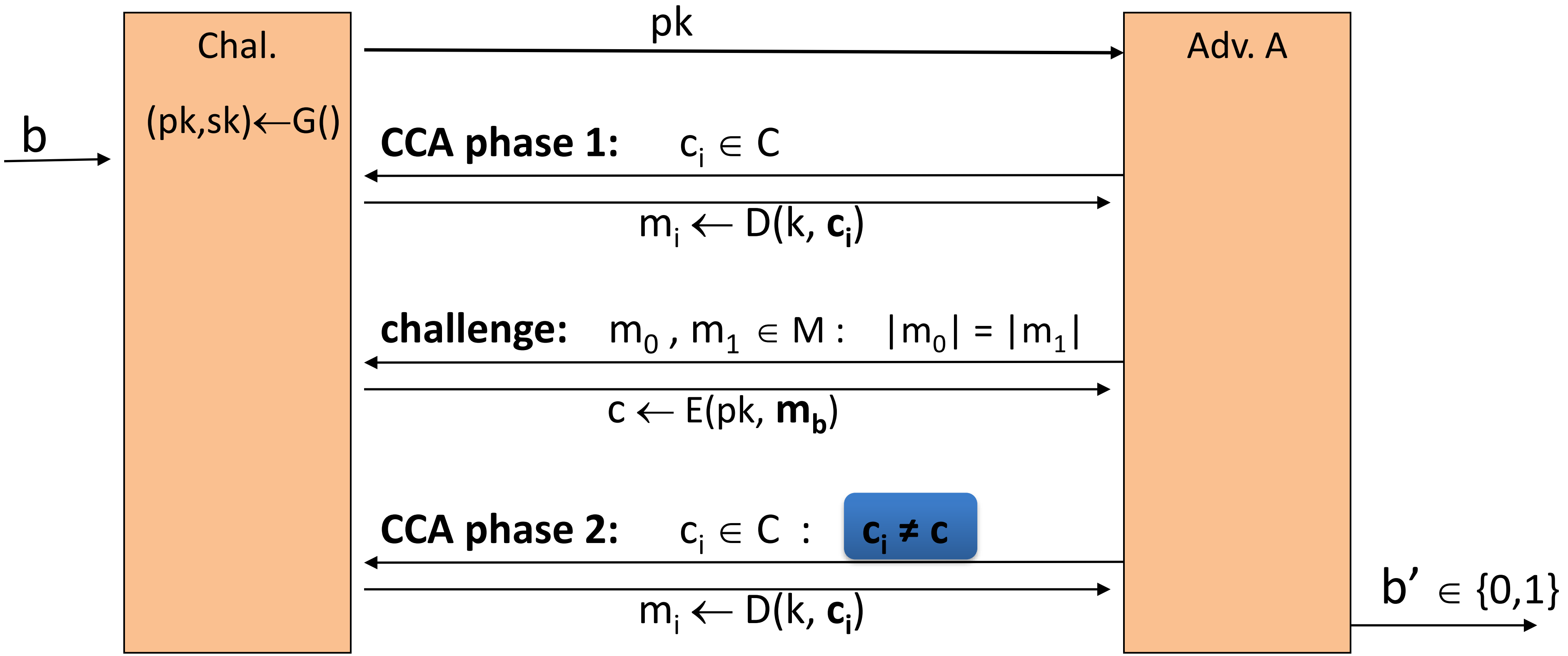
What if attacker can tamper with ciphertext?



Attacker is given decryption of msgs
that start with **"to: attacker"**

(pub-key) Chosen Ciphertext Security: definition

$\mathbb{E}$ = (G,E,D) public-key enc. over (M,C). For b=0,1 define EXP(b):

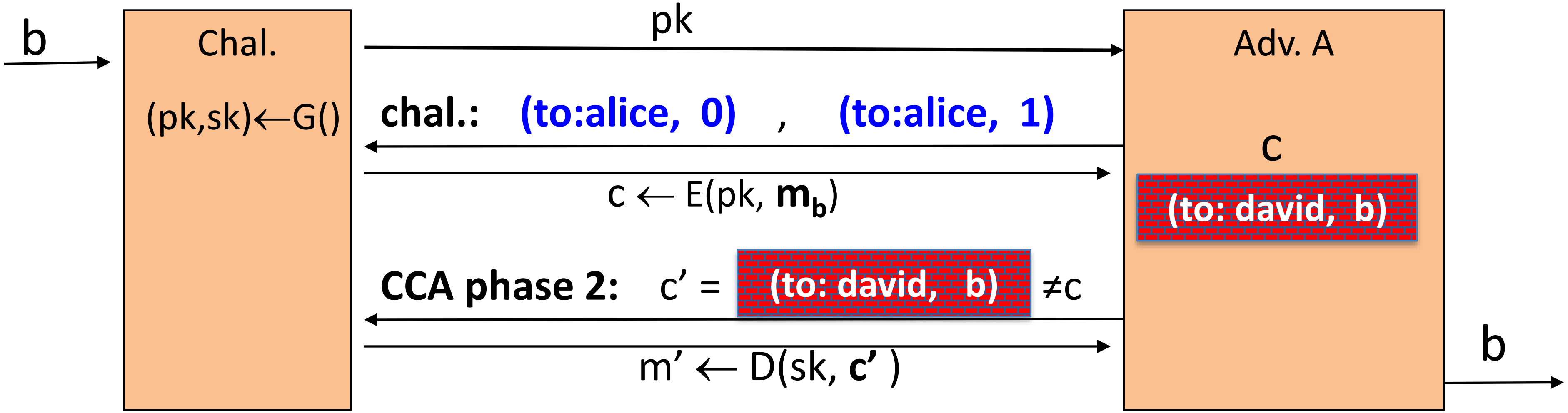


Chosen ciphertext security: definition

Def: $\mathbb{E}$ is CCA secure (a.k.a IND-CCA) if for all efficient A:

$$Adv_{CCA}[A,\mathbb{E}] = \Big| \Pr[EXP(0)=1] - \Pr[EXP(1)=1] \Big| \quad \text{is negligible.}$$

Example: Suppose (to: alice, body) ⟶ (to: david, body)



Active attacks: symmetric vs. pub-key

Recall: secure symmetric cipher provides **authenticated encryption**

[chosen plaintext security & ciphertext integrity]

- Roughly speaking: **attacker cannot create new ciphertexts**
- Implies security against chosen ciphertext attacks

In public-key settings:

- Attacker **can** create new ciphertexts using pk !!
- So instead: we directly require chosen ciphertext security

Trapdoor Permutations

Trapdoor functions (TDF)

<u>Def</u>: a trapdoor func. $X \longrightarrow Y$ is a triple of efficient algs. (G, F, F^{-1})

- $G()$: randomized alg. outputs a key pair (pk, sk)
- $F(pk,\cdot)$: det. alg. that defines a function $X \longrightarrow Y$
- $F^{-1}(sk,\cdot)$: defines a function $Y \longrightarrow X$ that inverts $F(pk,\cdot)$

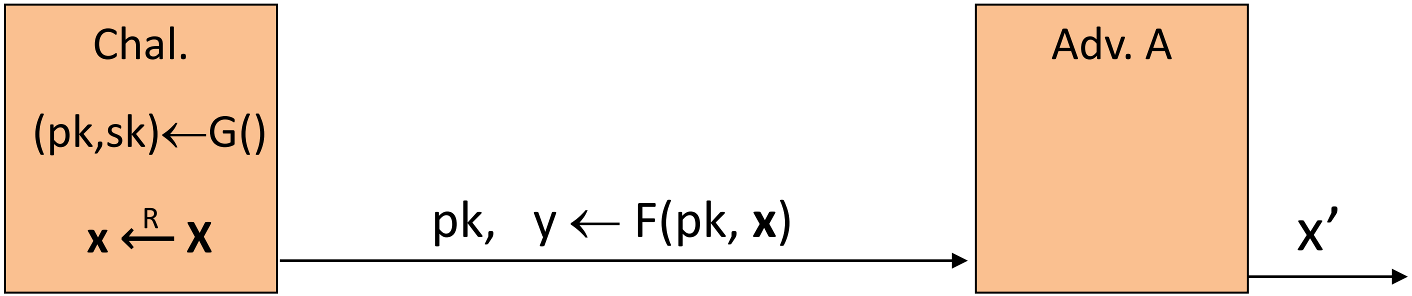
More precisely: $\forall (pk, sk)$ output by G

$$\forall x \in X: \quad F^{-1}(sk, F(pk, x)) = x$$

Secure Trapdoor Functions (TDFs)

(G, F, F^{-1}) is secure if $F(pk, \cdot)$ is a "one-way" function:

can be evaluated, but cannot be inverted without sk

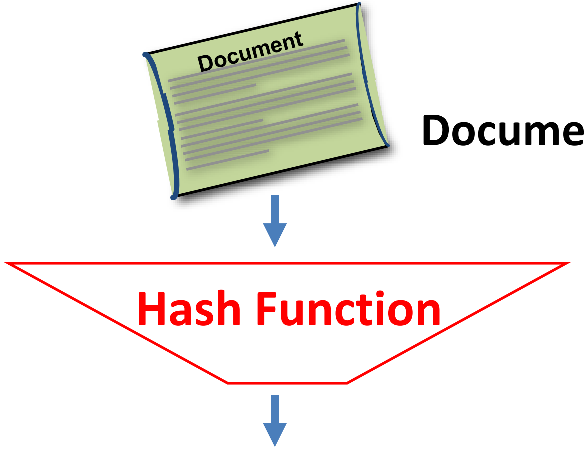


Def: (G, F, F^{-1}) is a secure TDF if for all efficient A:

$$\mathrm{Adv}_{OW}[A,F] = \mathbf{\Pr[\, x = x' \,]} < \text{negligible}$$

Hash Functions

- **Hash functions:**
  - **Input:** arbitrary length
  - **Output:** fixed length (generally much shortern than the input)



Document with arbitrary length

Hash value for the document
(fixed length, e.g. 256 bit)

One-Way Hash Algorithm

- A one-way hash algorithm hashes an input document into a condensed short output (say of 256 bits)
  - Denoting a one-way hash algorithm by H(.), we have:
    - Input: m - a binary string of any length
    - Output: H(m) - a binary string of L bits, called the “hash of m under H”.
    - The output length parameter L is fixed for a given one-way hash function H,
    - Examples:
      - The one-way hash function “MD5” has L = 128 bits
      - The one-way hash function “SHA-1” has L = 160 bits

Properties of One-Way Hash Algorithm

- A good one-way hash algorithm H needs to have these properties:
  - 1. **Easy to Evaluate**:
    - The hashing algorithm should be fast
  - 2. **Hard to Reverse**:
    - There is no feasible algorithm to "**reverse**" a hash value,
    - That is, given any hash value **h**, it is computationally infeasible to find any document **m** such that **H(m) = h**.
  - 3. **Hard to find Collisions**:
    - There is no feasible algorithm to find two or more input documents which are hashed into the same condensed output,
    - That is, it is computationally infeasible to find any two documents **m1**, **m2** such that **H(m1)= H(m2)**.
  - 4. **A small change** to a message **should change the hash value so extensively** that the new hash value appears uncorrelated with the old hash value

Public-key encryption from TDFs

- (G, F, F^{-1}): secure TDF $X \longrightarrow Y$
- (E_s, D_s) : symmetric auth. encryption defined over (K,M,C)
- $H: X \longrightarrow K$ a hash function

We construct a pub-key enc. system (G, E, D):

Key generation G: same as G for TDF

Public-key encryption from TDFs

- (G, F, F^{-1}): secure TDF $X \longrightarrow Y$
- (E_s, D_s) : symmetric auth. encryption defined over (K,M,C)
- $H: X \longrightarrow K$ a hash function

E(pk, m) :

$x \xleftarrow{R} X$, $y \longleftarrow F(pk, x)$

$k \longleftarrow H(x)$, $c \longleftarrow E_s(k, m)$

output (y, c)

D(sk, (y,c)) :

$x \longleftarrow F^{-1}(sk, y)$,

$k \longleftarrow H(x)$, $m \longleftarrow D_s(k, c)$

output m

In pictures:

$$\underbrace{F(pk, x)}_{\text{header}} \; \underbrace{E_s(\, H(x), \; m \,)}_{\text{body}}$$

Security Theorem:

If **(G, F, F^{-1})** is a secure TDF, **(E_s, D_s)** provides auth. enc.

and **H:** $X \longrightarrow K$ is a "random oracle"

then **(G,E,D)** is CCA^{ro} secure.

Incorrect use of a Trapdoor Function (TDF)

Never encrypt by applying F directly to plaintext:

E(pk, m) :

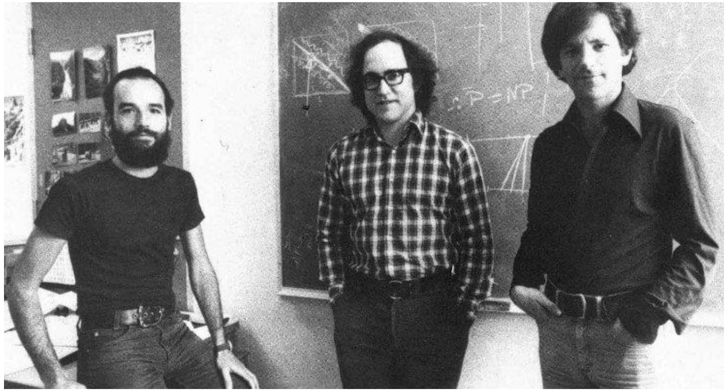
output $c \longleftarrow F(pk, m)$

D(sk, c) :

output $F^{-1}(sk, c)$

Problems:

- Deterministic: cannot be semantically secure !!
- Many attacks exist (next segment)



The RSA trapdoor permutation

- One of the first practical responses to the challenge posed by Diffie-Hellman was developed by ***R****on Rivest*, *Adi* ***S****hamir*, and *Len* ***A****dleman* of MIT in 1977
- Resulting algorithm is known as *RSA*
- Based on properties of *prime numbers* and results from *number theory*

Review: trapdoor permutations

Three algorithms: (G, F, F^{-1})

- G: outputs pk, sk. pk defines a function $F(pk, \cdot): X \to X$
- $F(pk, x)$: evaluates the function at x
- $F^{-1}(sk, y)$: inverts the function at y using sk

Secure trapdoor permutation:

The function $F(pk, \cdot)$ is one-way without the trapdoor sk

Review: arithmetic mod composites

Let $N = p \cdot q$ where p,q are prime where $p,q \approx N^{1/2}$

$Z_N = \{0,1,2,\ldots,N-1\}$; $(Z_N)^*$ = {invertible elements in Z_N}

Facts: $x \in Z_N$ is invertible $\Leftrightarrow$ $\gcd(x,N) = 1$

– Number of elements in $(Z_N)^*$ is $\varphi(N) = (p-1)(q-1) = N-p-q+1$

Euler's thm: $\boxed{\forall\, x \in (Z_N)^* \; : \; x^{\varphi(N)} = 1}$

The RSA trapdoor permutation

First published: Scientific American, Aug. 1977.

Very widely used:

- SSL/TLS: certificates and key-exchange
- Secure e-mail and file systems

… many others

The RSA trapdoor permutation

G(): choose random primes p,q ≈1024 bits. Set **N=pq**.

choose integers **e , d** s.t. **e·d = 1 (mod φ(N))**

output pk = (N, e) , sk = (N, d)

F(pk, x): $\mathbb{Z}_N^* \rightarrow \mathbb{Z}_N^*$; **RSA(x) = x^e^** (in Z_N)

$\mathbf{F^{-1}}$**(sk, y)** $= y^d$; $y^d = \mathbf{RSA(x)^d} = x^{ed} = x^{k\varphi(N)+1} = \left(x^{\varphi(N)}\right)^k \cdot \mathbf{x} = x$

RSA - small example

- Bob (**keys generation**):
  - chooses 2 primes: **p=5, q=11**
  - multiplies p and q: $\mathbf{n = p \times q = 55}$
  - chooses a number **e=3** s.t. **gcd(e, 40) = 1**
  - compute **d=27** that satisfy $\mathbf{(3 \times d) \bmod 40 = 1}$

  - **Bob's public key: (3, 55)**
  - **Bob's private key: 27**

RSA - small example

- Alice (**encryption**):
  - has a message **m=13** to be sent to Bob
  - finds out **Bob's public encryption key (3, 55)**
  - calculates **c** as follows:

$$
\begin{aligned}
c &= m^e \bmod n \\
&= 13^3 \bmod 55 \\
&= 2197 \bmod 55 \\
&= 52
\end{aligned}
$$

  - sends the ciphertext **c=52** to Bob

RSA - small example

- Bob (**decryption**):
  - receives the ciphertext **c=52** from Alice
  - uses his matching private decryption key **27** to calculate **m**:

$$m = 52^{27} \bmod 55$$
$$= 13 \text{ (Alice's message)}$$

The RSA assumption

RSA assumption: RSA is one-way permutation

For all efficient algs. A:

$$\Pr\Big[\ A(N,e,y) = y^{1/e}\ \Big] < \text{negligible}$$

where $p,q \xleftarrow{R}$ n-bit primes, $N \leftarrow pq$, $y \xleftarrow{R} Z_N^{\ *}$

Review: RSA pub-key encryption (ISO std)

(E_s, D_s): symmetric enc. scheme providing auth. encryption.

H: $Z_N \to K$ where K is key space of (E_s, D_s)

- **G**(): generate RSA params: pk = (N,e), sk = (N,d)
- **E**(pk, m): (1) choose random x in Z_N

  (2) $y \leftarrow RSA(x) = x^e$, $k \leftarrow H(x)$

  (3) output $(y\ ,\ E_s(k,m)\ )$
- **D**(sk, (y, c)): output $D_s\Big(\ H\big(RSA^{-1}(y)\big)\ ,\ c\Big)$ -> m

Textbook RSA is insecure

Textbook RSA encryption:

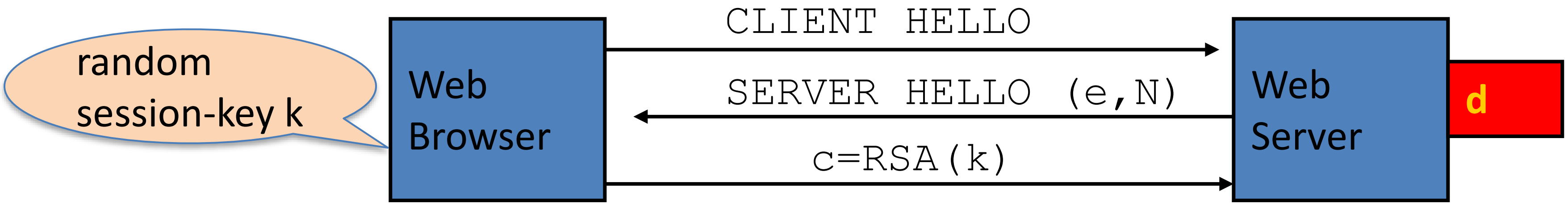
- public key: **(N,e)** Encrypt: $\mathbf{c \longleftarrow m^e}$ (in Z_N)
- secret key: **(N,d)** Decrypt: $\mathbf{c^d \longrightarrow m}$

Insecure cryptosystem !!

- Is not semantically secure and many attacks exist

⇒ The RSA trapdoor permutation is not an encryption scheme !

A simple attack on textbook RSA



Suppose k is 64 bits: $k \in \{0,\ldots,2^{64}\}$. Eve sees: $c = k^e$ in Z_N

If $\mathbf{k = k_1 \cdot k_2}$ where $k_1, k_2 < 2^{34}$ (prob. $\approx$20%) then $\mathbf{c/k_1^{\,e} = k_2^{\,e}}$ in Z_N

Meet-in-the-middle attack:

Step 1: build table: $c/1^e, c/2^e, c/3^e, \ldots, c/2^{34e}$. time: 2^{34}

Step 2: for $k_2 = 0,\ldots, 2^{34}$ test if $k_2^{\,e}$ is in table. time: 2^{34}

Output matching (k_1, k_2). Total attack time: $\approx 2^{40} << 2^{64}$

Is RSA a one-way function?

Is it really hard to invert RSA without knowing the trapdoor?

Is RSA a one-way permutation?

To invert the RSA one-way func. (without d) attacker must compute:

$$x \quad \text{from} \quad c = x^e \pmod{N}.$$

How hard is computing e'th roots modulo N ??

Best known algorithm:

- Step 1: factor N (hard)
- Step 2: compute e'th roots modulo p and q (easy)

Shortcuts?

Must one factor N in order to compute e'th roots?

To prove no shortcut exists show a reduction:

- Efficient algorithm for e'th roots mod N

  $\Rightarrow$ efficient algorithm for factoring N.

- Oldest problem in public key cryptography.

Some evidence no reduction exists: (BV'98)

- "Algebraic" reduction $\Rightarrow$ factoring is easy.

How **not** to improve RSA's performance

To speed up RSA decryption use small private key d ($d \approx 2^{128}$)

$$\boxed{c^d = m \pmod{N}}$$

Wiener'87: if $d < N^{0.25}$ then RSA is insecure.

BD'98: if $d < N^{0.292}$ then RSA is insecure (open: $d < N^{0.5}$)

<u>Insecure:</u> priv. key d can be found from (N,e)

Wiener's attack

(N,e) => d and d < $N^{0.25}/3$

Recall: e·d = 1 (mod φ(N)) $\Rightarrow$ $\exists$ k$\in$Z : e·d = k·φ(N) + 1

$$\left|\frac{e}{\psi(N)} - \frac{k}{d}\right| = \frac{1}{d \cdot \varphi(N)} \leq \frac{1}{\sqrt{N}}$$

φ(N) = N-p-q+1 $\Rightarrow$ |N − φ(N)| $\leq$ p+q $\leq$ 3 $\sqrt{N}$

d $\leq$ $N^{0.25}/3$ $\Rightarrow$ $\frac{1}{2d^2} - \frac{1}{\sqrt{N}} \geq \frac{3}{\sqrt{N}}$

$$\left|\frac{\mathrm{e}}{N} - \frac{k}{d}\right| \leq \left|\frac{\mathrm{e}}{N} - \frac{\mathrm{e}}{\varphi(N)}\right| + \left|\frac{\mathrm{e}}{\varphi(N)} - \frac{k}{d}\right| \leq \frac{1}{2d^2}$$

Continued fraction expansion of e/N gives k/d.

e·d = 1 (mod k) $\Rightarrow$ gcd(d,k)=1 $\Rightarrow$ can find d from k/d

RSA in Practice

RSA With Low public exponent

To speed up RSA encryption use a small e: $c = m^e \pmod{N}$

- Minimum value: **$e=3$** ($\gcd(e, \varphi(N)) = 1$) (Q: why not 2?)
- Recommended value: **$e=65537=2^{16}+1$**

Encryption: 17 multiplications

Asymmetry of RSA: fast enc. / slow dec.

- ElGamal (next week): approx. same time for both.

Key lengths

Security of public key system should be comparable to security of symmetric cipher:

Cipher key-size	RSA Modulus size
80 bits	1024 bits
128 bits	3072 bits
256 bits (AES)	**15360** bits

Implementation attacks

Timing attack: [Kocher et al. 1997] , [BB'04]

The time it takes to compute $c^d \pmod{N}$ can expose d

Power attack: [Kocher et al. 1999)

The power consumption of a smartcard while it is computing $c^d \pmod{N}$ can expose d.

Faults attack: [BDL'97]

A computer error during $c^d \pmod{N}$ can expose d.

A common defense: check output. 10% slowdown.

An Example Fault Attack on RSA (CRT)

A common implementation of RSA decryption: $x = c^d$ in Z_N

$$\left.\begin{array}{ll} \text{decrypt mod } p: & x_p = c^d \text{ in } Z_p \\ \\ \text{decrypt mod } q: & x_q = c^d \text{ in } Z_q \end{array}\right\} \text{combine to get } x = c^d \text{ in } Z_N$$

Suppose error occurs when computing x_q, but no error in x_p

Then: output is x' where $x' = c^d$ in Z_p but $x' \neq c^d$ in Z_q

$\Rightarrow$ $(x')^e = c$ in Z_p but $(x')^e \neq c$ in Z_q $\Rightarrow$ $\gcd\big((x')^e - c, N\big) = \boxed{}$

RSA Key Generation Trouble [Heninger et al./Lenstra et al.]

OpenSSL RSA key generation (abstract):

```
prng.seed(seed)
p = prng.generate_random_prime()
prng.add_randomness(bits)
q = prng.generate_random_prime()
N = p*q
```

Suppose poor entropy at startup:

- Same p will be generated by multiple devices, but different q
- N_1 , N_2 : RSA keys from different devices $\Rightarrow$ $\gcd(N_1,N_2) = p$

RSA Key Generation Trouble [Heninger et al./Lenstra et al.]

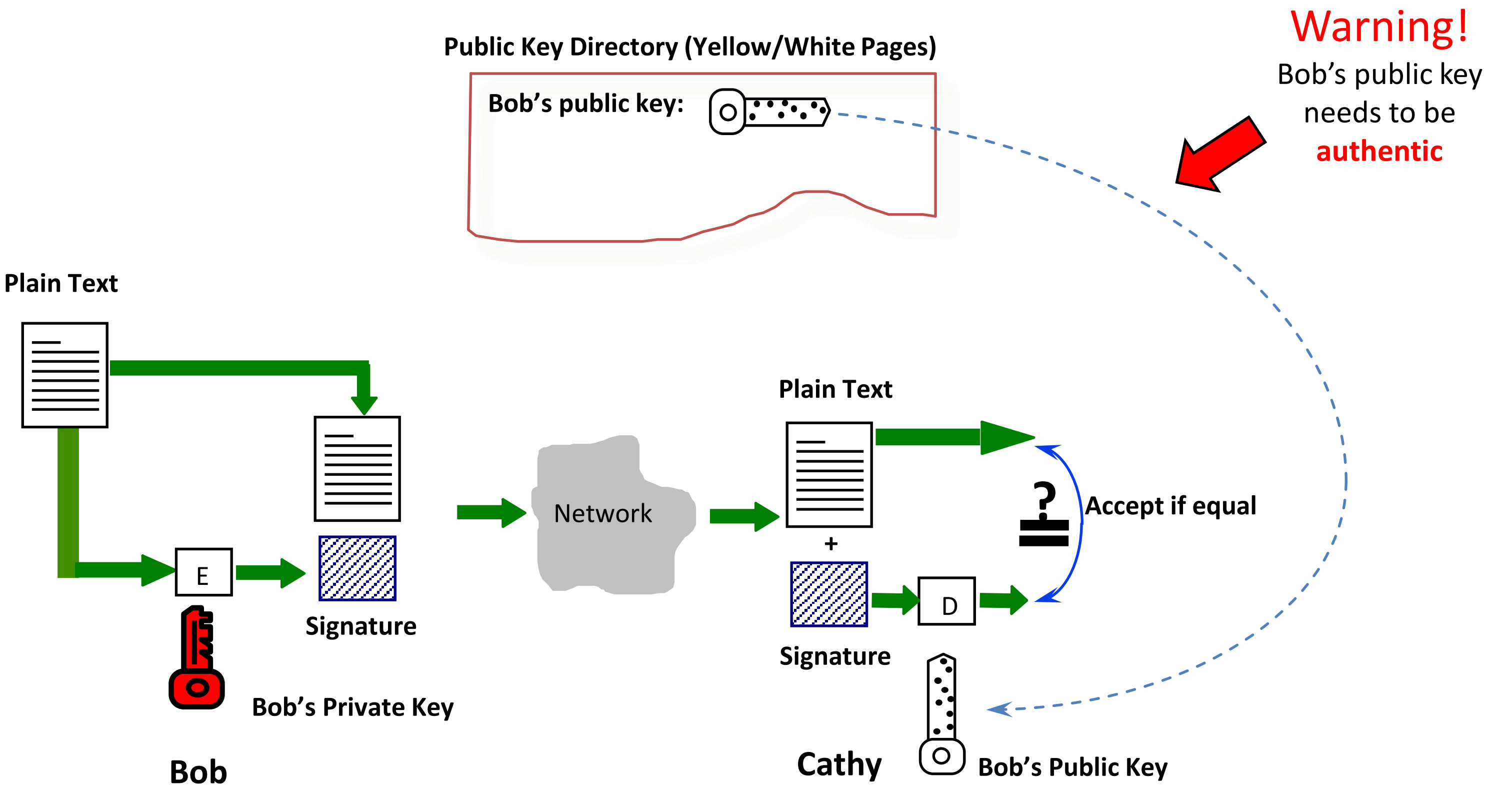
Experiment: factors 0.4% of public HTTPS keys !!

Lesson:

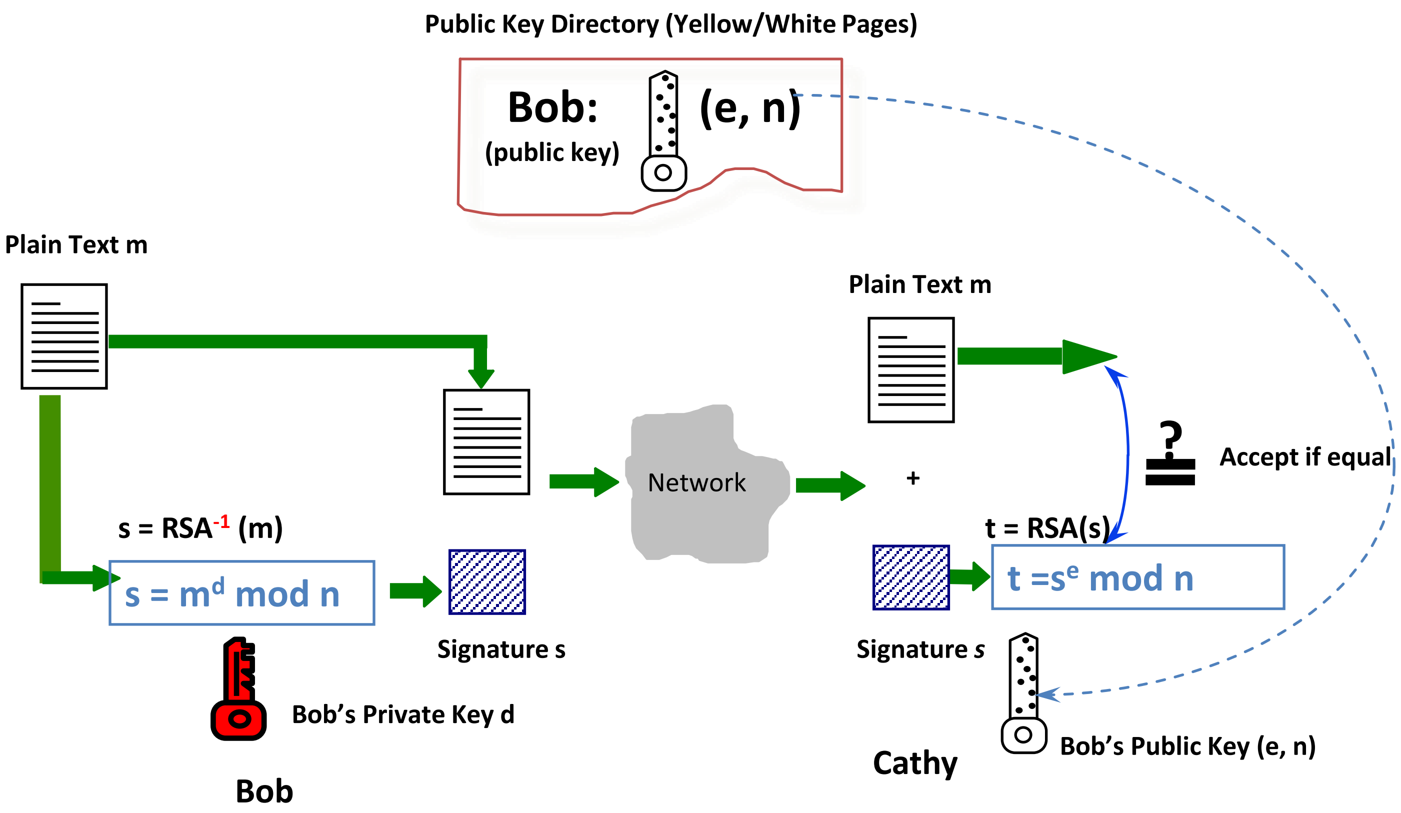
- Make sure random number generator is properly seeded when generating keys

Digital Signatures

Digital Signature



Digital Signature (based on RSA)



RSA Signature - small example

- Bob (**keys generation**):
  - chooses 2 primes: **p=5, q=11**
  - multiplies p and q: $\mathbf{n = p \times q = 55}$
  - chooses a number **e=3** s.t. **gcd(e, 40) = 1**
  - compute **d=27** that satisfy $\mathbf{(3 \times d) \bmod 40 = 1}$

  - **Bob's public key: (3, 55)**
  - **Bob's private key: 27**

RSA Signature - small example

- Bob:
  - has a document **m=19** to sign:
  - uses **his private key** **d=27** to **calculate the digital signature** of **m=19**:

$$\begin{aligned} s &= m^d \bmod n \\ &= 19^{27} \bmod 55 \\ &= 24 \end{aligned}$$

  - **appends 24 to 19**.

    Now **(m, s) = (19, 24**) indicates that the **doc is 19**, and **Bob's signature on the doc is 24**.

RSA Signature - small example

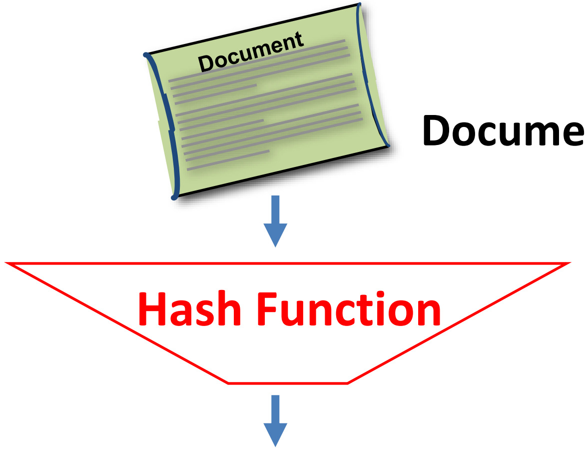
- Cathy, a verifier:
  - **receives** a pair **(m,s)=(19, 24)**
  - looks up the phone book and **finds out Bob's public key** (e, n)=(3, 55)
  - **calculates** $\mathbf{t} = s^e \bmod n$
    $= 24^3 \bmod 55$
    $= 19$
  - **checks whether t=m**
  - **confirms that (19,24) is a genuinely signed document of Bob if t=m.**

How about Long Documents ?

- In the previous example, a document has to be an integer in [0,...,n)
- To sign a very long document, we need a so called one-way **hash algorithm**
- **Instead of signing directly on a doc**,
  - we **hash the doc first**,
  - and **sign the hashed data** which is normally short.

Hash Functions

- **Hash functions:**
  - **Input:** arbitrary length
  - **Output:** fixed length (generally much shortern than the input)

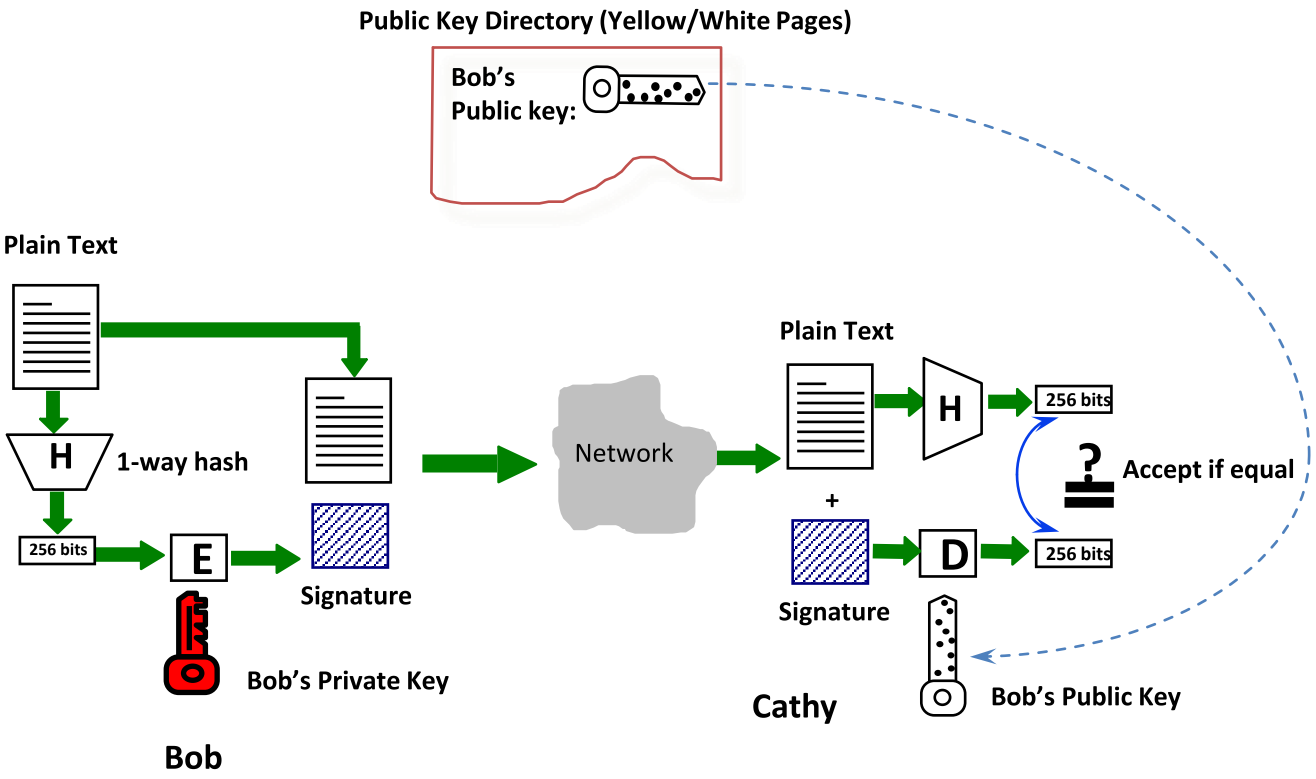


Document with arbitrary length

Hash value for the document
(fixed length, e.g. 256 bit)

Rather than signing the original document, we **sign its hash value**

Digital Signature (for long docs)



Why Digital Signature ?

- Unforgeable
  - takes 1 billion years to forge !
- Un-deniable by the signatory
- Universally verifiable
- Differs from doc to doc

Digital Signature - summary

- Three (3) steps are involved in digital signature
  - Setting up public and private keys
  - Signing a document
  - Verifying a signature

Setting up Public & Private Keys

- Bob does the following
  - prepares a pair of public and private keys
  - Publishes his public key in the public key file (such as an on-line phone book)
  - Keeps the private key to himself
- Note:
  - Setting up needs only to be done **once** !

Signing a Document

- Once setting up is completed, Bob can sign a document (such as a contract, a cheque, a certificate, ...) using the private key
- The pair of document & signature is a proof that Bob has signed the document.

Verifying a Signature

- Any party, say Cathy, can verify the pair of document and signature, by using Bob's public key in the public key file.
- Important !
  - Cathy does NOT have to have public or private key !

(Other) Asymmetric Cryptosystems

ElGamal Cryptosystem

Encryption schemes built from the Diffie-Hellman protocol

- **Key Generation** (for Bob)
  - chooses a prime **p** and a number **g** *primitive root modulo p*
    - i.e., for every integer **a** coprime to **p**, there is an interger **k** such that $\mathbf{g^k = a \bmod p}$
      - Two integers are coprime if their gcd is 1
  - chooses a random exponent **a** in [0, p-2]
  - computes $\mathbf{A = g^a \bmod p}$
  - **public key** (published in the phone book): **(p,g,A)**
  - **private key**: **a**

ElGamal Cryptosystem

- **Encryption:** Alice has a message **m** ($0<=m<n$) to be sent to Bob:
  - finds out **Bob's public key (p,g,A)**.
  - chooses a random exponent **b** in [0,p-2]
  - computes $\mathbf{B = g^b \bmod p}$
  - computes $\mathbf{c = A^b m \bmod p}$.
  - The complete ciphertex is **(B,c)**
  - sends the ciphertext **(B,c)** to Bob.

ElGamal Cryptosystem

- **Decryption**: Bob
  - receives the ciphertext **(B,c)** from Alice.
  - uses his matching private decryption key a to calculate **m** as follows.
    - Compute $\mathbf{x = p\text{-}1\text{-}a}$
    - Compute $\mathbf{m = B^x\, c \bmod p}$

ElGamal Cryptosystem

- Randomized cryptosystem
- Based on the Diffie–Hellman key exchange
- Efficiency
  - The ciphertext is twice as long as the plaintext. This is called message expansion and is a disadvantage of this cryptosystem.
- Security
  - Its security depends upon the difficulty of a certain problem related to computing discrete logarithms.

Rabin Cryptosystem

Key Generation (for Bob)

- generates 2 large random and distinct primes **p, q** s.t.

  $p \pmod 4 = q \pmod 4 = 3$ (other options are possible, this makes decryption more efficient)

- multiplies p and q: $\mathbf{n = p \times q}$
- **public key** (published in the phone book): **n**
- **private key**: **(p, q)**

Rabin Cryptosystem

- **Encryption:** Alice has a message **m** ($0<=m<n$) to be sent to Bob:
  - finds out **Bob's public key** **n**.
  - calculates the ciphertext **$c= m^2 \bmod n$**.
  - sends the ciphertext **c** to Bob.

Rabin Cryptosystem

- **Decryption:** Bob
  - receives the ciphertext **c** from Alice.
  - uses his matching private decryption key **(p,q)** to calculate **m** as follows.
    - Compute $m_p = c^{(p+1)/4} \bmod p$
    - Compute $m_q = c^{(q+1)/4} \bmod q$
    - Find y_p and y_q such that $y_p\, p + y_q\, q = 1$ (Euclidean algorithm)
    - Compute $r = (y_p\, p\, m_q + y_q\, q\, m_p) \bmod n$
    - Compute $s = (y_p\, p\, m_q - y_q\, q\, m_p) \bmod n$
    - One of $r, -r, s, -s$ must be the original message **m**

Rabin Cryptosystem

- Efficiency
  - Encryption more efficient than RSA encryption

- Security
  - The Rabin cryptosystem has the advantage that the problem on which it relies has been proved to be as hard as integer factorization
    - Recovering the plaintext ***m*** from the ciphertext ***c*** and the public key ***n*** is computationally equivalent to factoring
    - Not currently known to be true for the RSA problem.