(1)

$$f(x) = \ln \frac{x^2+4}{x}$$

$$D(f) = \left\{ x \;\middle|\; \frac{x^2+4}{x} > 0 \right\} =$$

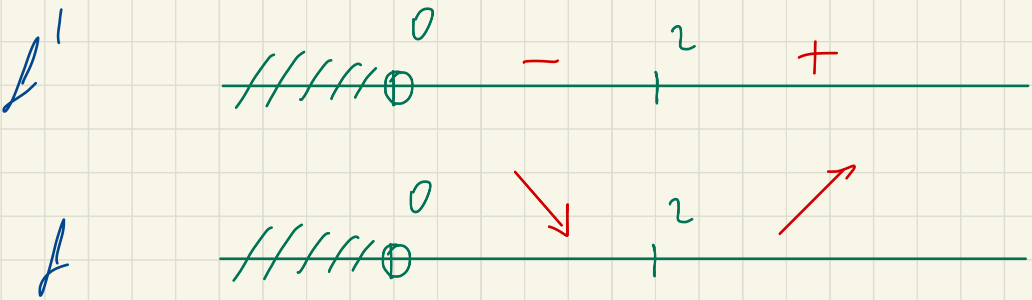
$$= \{ x \mid x > 0 \}$$

$$\lim_{x \to 0^+} \ln \underbrace{\frac{x^2+4}{x}}_{+\infty} = +\infty$$

$$\lim_{x \to +\infty} \ln \frac{x^2+4}{x} = +\infty$$

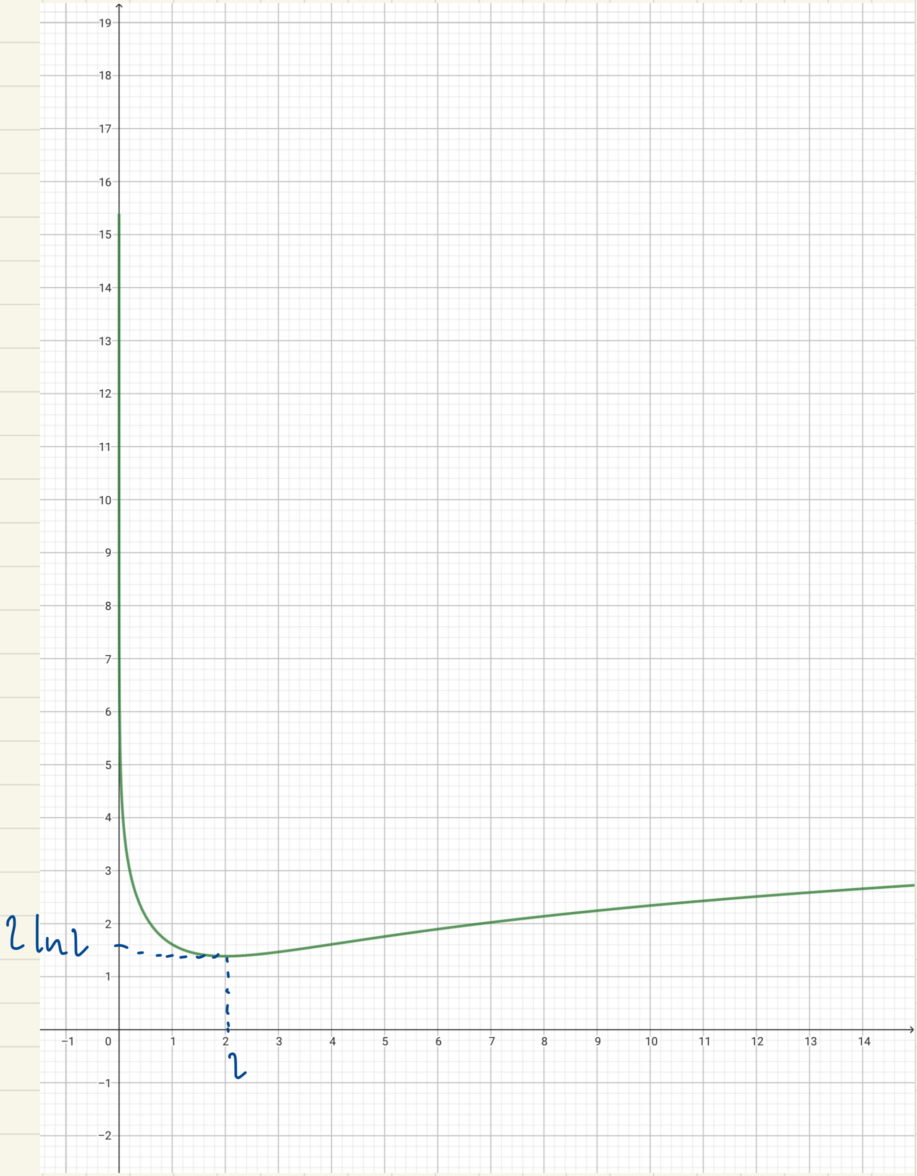
$$f'(x) = \frac{x}{x^2+4} \cdot \frac{2x^2 - x^2 - 4}{x^2} =$$

$$= \underbrace{\frac{1}{(x^2+4)\cdot x}}_{>0} \cdot (x^2 - 4)$$



$$f(2) = \ln \frac{4+4}{2} = \ln 4 = 2 \ln 2$$

p. di MIN. LOCALE



$\operatorname{Im} f = [2\ln 2, +\infty[$

$f(x) = k$ ha 1 soluzione se $k = 2\ln 2$

2

$$\sqrt{1+\left(-2x^2+\frac{2}{3}x^4\right)} =$$

$$= 1 + \frac{1}{2}\left(-2x^2+\frac{2}{3}x^4\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(-2x^2+\frac{2}{3}x^4\right)^2 + o(x^4)$$

$$= 1 - x^2 + \frac{1}{3}x^4 - \frac{1}{\cancel{8}\,2}\cancel{4}x^4 + o(x^4) =$$

$$= 1 - x^2 - \frac{1}{6}x^4 + o(x^4)$$

$$\cos\left(2x+x^3\right) = 1 - \frac{1}{2}\left(2x+x^3\right)^2 +$$

$$+ \frac{1}{4!}\left(2x+x^3\right)^4 + o(x^4) =$$

$$= 1 - \frac{1}{2}\left(4x^2+4x^4+\cancel{x^6}\right) + \frac{1}{24}\cdot 16x^4 + o(x^4)$$

$$= 1 - 2x^2 - 2x^4 + \frac{2}{3}x^4 + o(x^4)$$

$$= 1 - 2x^2 - \frac{4}{3}x^4 + o(x^4)$$

$$\sqrt{1+\left(-2x^2+\frac{2}{3}x^4\right)} - \cos\left(2x+x^3\right) - x^2 =$$

$$= \cancel{1} - \cancel{x^2} - \frac{1}{6}x^4 + o(x^4) -$$

$$- \cancel{1} + \cancel{2x^2} + \frac{4}{3}\cdot x^4 - \cancel{x^2} =$$

$$= \frac{7}{6}x^4 + o(x^4) = \frac{7}{6}x^4 + o(x^4)$$

$$\lim_{x\to 0} \frac{\sqrt{1+\left(-2x^2+\frac{2}{3}x^4\right)} - \cos\left(2x+x^3\right) - x^2}{x^4}$$

$$= \frac{7}{6}$$