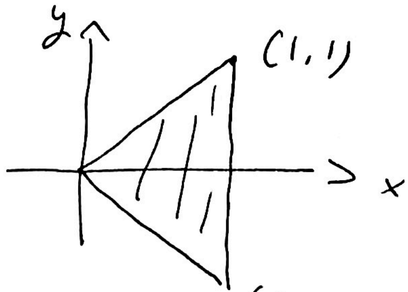
ES 2

$$\int_T \sqrt{\frac{1+2x}{x+y+1}}\,dx\,dy = \left(\begin{matrix}1+2x>0 \quad e\\ x+y+1>0\\ su\ T\end{matrix}\right)$$



$$= \int_T \frac{\sqrt{1+2x}}{\sqrt{x+y+1}}\,dx\,dy = \int_0^1 \sqrt{1+2x}\left(\int_{-x}^{x}\frac{dy}{\sqrt{x+y+1}}\right)dx$$

$$= \int_0^1 dx\,\sqrt{1+2x}\left[2\sqrt{x+y+1}\right]_{y=-x}^{y=x} = \int_0^1 2\sqrt{1+2x}\left(\sqrt{1+2x}-1\right)dx$$

$$= \int_0^1 2(1+2x)\,dx - 2\int_0^1 \sqrt{1+2x}\,dx = \left[2x+2x^2-\frac{2}{3}(1+2x)^{3/2}\right]_0^1$$

$$= 2+2-\frac{2}{3}\cdot 3^{3/2}-\left(0+0-\frac{2}{3}\right) = 4-2\sqrt{3}+\frac{2}{3}$$

ES 1 $\quad f(x,y) = (x^2+y^2+1)^2 - 9(x+y)^2$

$$\begin{cases}\partial_x f = 4x(x^2+y^2+1) - 18(x+y) = 0\\ \partial_y f = 4y(x^2+y^2+1) - 18(x+y) = 0\end{cases}$$

Sottraggo le equazioni $\Rightarrow 4(x-y)(x^2+y^2+1) = 0 \Rightarrow x=y$

$$\Rightarrow \begin{cases}x=y\\ 4y(x^2+y^2+1) - 18(x+y) = 0\end{cases} \Rightarrow \begin{cases}x=y\\ 4x(2x^2+1) - 36x = 0\end{cases}$$

$$\begin{cases}x=y\\ 4x(2x^2+1-9) = 0\end{cases} \Rightarrow \begin{cases}x=y\\ 8x(x^2-4) = 0\end{cases}$$

P. critici $\quad (0,0),\ (2,2),\ (-2,-2)$

$$\partial_{xx} f = 4(x^2+y^2+1) + 8x^2 - 18 = 12x^2+4y^2-14 = 2(6x^2+2y^2-7)$$

$$\partial_{yy} f = 2(6y^2+2x^2-7)$$

$$\partial_{xy} f = 8xy - 18 = 2(4xy-9)$$

Dunque

$$Hf(0,0) = \begin{bmatrix} -14 & -18 \\ -18 & -14 \end{bmatrix}$$ che ha $\det < 0 \Rightarrow (0,0)$ è di sella

$$Hf(2,2) = \begin{bmatrix} 50 & 14 \\ 14 & 50 \end{bmatrix} = Hf(-2,-2)$$ visto che $\begin{cases} 50 > 0 \\ 50^2 - 14^2 > 0 \end{cases}$

$(2,2)$ e $(-2,-2)$ sono di minimo.

Scrivo eq. piano tangente in $(1,-1)$.

$f(1,-1) = 3^2 - 9 \cdot 0 = 9$

$\nabla f(1,-1) = (12, -12)$

Dunque l'equazione richiesta è

$$z = 9 + \langle (12,-12), (x-1, y+1) \rangle = 9 + 12(x-1) - 12(y+1)$$