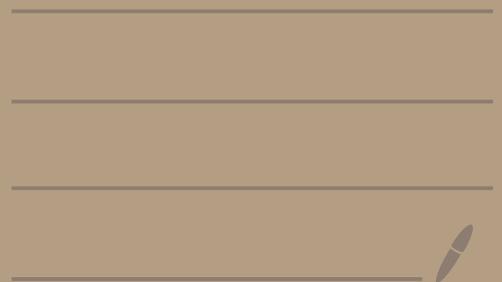


# Soluzioni 16 luglio 2021

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ESERCIZIO (3) :

$$f(x) = e^{\frac{x^2+9}{x^2-9}}$$

$$\begin{aligned} \textcircled{1} \quad \text{D}(f) &= \{x \in \mathbb{R} \mid x^2 - 9 \neq 0\} \\ &= \mathbb{R} \setminus \{\pm 3\} \end{aligned}$$

$$\textcircled{2} \quad f(-x) = e^{\frac{(-x)^2+9}{(-x)^2-9}} = e^{\frac{x^2+9}{x^2-9}} = f(x)$$

$\Rightarrow f$  è pari  $\Rightarrow$  il grafico  
di  $f$  è simmetrico rispetto

l'asse delle  $y$ .

Possiamo studiare  $f$  solo per  $x \geq 0$



$$f(0) = e^{-1}$$

$$\lim_{x \rightarrow 3^-} e^{\frac{x^2+9}{x^2-9}} = 0^+$$

$$\left( \lim_{x \rightarrow 3^-} \frac{x^2+9}{x^2-9} = \frac{-\infty}{+\infty} \right)$$

Diagram illustrating the limit process. A parabola  $y = x^2 - 9$  is shown with x-intercepts at -1 and 1. A red '3+' is written below the x-axis, with an arrow pointing to the right side of the parabola. The expression  $\frac{x^2+9}{x^2-9}$  is shown with the numerator  $x^2+9$  circled and an arrow pointing to the value 2. The denominator  $x^2-9$  is also circled, with an arrow pointing to the value 0<sup>-</sup>.

$$\lim_{x \rightarrow 3^+} e^{\frac{x^2+9}{x^2-9}} = +\infty$$

$$\lim_{x \rightarrow +\infty} e^{\frac{x^2+9}{x^2-9}} = \lim_{x \rightarrow +\infty} e^{\frac{1+\frac{9}{x^2}}{1-\frac{9}{x^2}}} = e$$

4

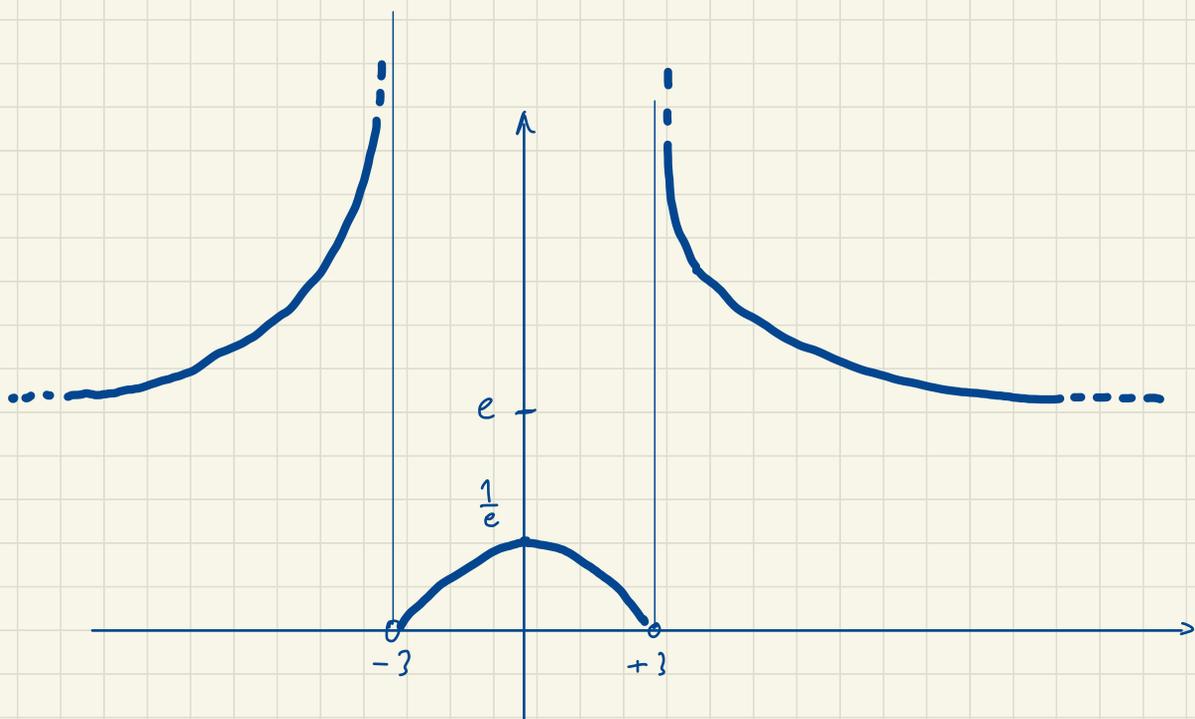
$$f(x) = e^{\frac{x^2+9}{x^2-9}}$$

$$f'(x) = e^{\frac{x^2+9}{x^2-9}} \cdot \frac{2x(x^2-9) - 2x(x^2+9)}{(x^2-9)^2} =$$

$$= e^{\frac{x^2+9}{x^2-9}} \cdot \frac{2x^3 - 18x - 2x^3 - 18x}{(x^2-9)^2} =$$

$$= \frac{e^{\frac{x^2+9}{x^2-9}}}{(x^2-9)^2} \cdot (-36x) \leq 0 \quad \text{für } x \geq 0$$
$$< 0 \quad \text{für } x > 0$$





$$\text{Im } f = ]0, \frac{1}{e}] \cup ]e, +\infty[$$

$$\# S = 2 \quad \text{rc } \subseteq ]0, \frac{1}{e}] \cup ]e, +\infty[$$

2

$$\lim_{x \rightarrow 0} \frac{\sin(\sin x) - \sin x + \frac{x^3}{6}}{x^5}$$

$$\sin x \approx x$$



lo teniamo così!

$$\sin(\sin x) = \sin x - \frac{\sin^3 x}{6} + \frac{\sin^5 x}{5!} + o(x^5)$$

to remember too!

$$\sin(\sin x) = \sin x - \frac{\sin^3 x}{6} +$$

$$+ \frac{\sin^5 x}{5!} + o(x^5)$$

$$= \sin x - \frac{1}{6} \left( x - \frac{x^3}{6} \right)^3 + \frac{x^5}{5!} + o(x^5) =$$

$$= \sin x - \frac{1}{6} \left( x^3 + 3 \cdot x^2 \cdot \left( -\frac{x^3}{6} \right) \right) +$$

$$+ \frac{x^5}{60} + o(x^5)$$

$$= \sin x - \frac{x^3}{6} + \frac{x^5}{12} + \frac{x^5}{120} + o(x^5)$$

$$= \sin x - \frac{x^3}{6} + \frac{11}{120} x^5 + o(x^5)$$

$\Rightarrow$

$$\sin(\sin x) - \sin x = -\frac{x^3}{6} + \frac{11}{120} x^5 + o(x^5)$$

$$\lim_{x \rightarrow 0} \frac{\sin(\sin x) - \sin x + \frac{x^3}{6}}{x^5} =$$

$$= \lim_{x \rightarrow 0} \frac{-\cancel{\frac{x^3}{6}} + \frac{11}{120} x^5 + o(x^5) + \cancel{\frac{x^3}{6}}}{x^5} =$$

$$\Rightarrow \lim_{x \rightarrow 0} \left( \frac{11}{120} + \frac{o(x^5)}{x^5} \right) = \frac{11}{120}$$