Soluzioni 16 luglio 2021

ESERCIZIO 3 :

$$f(x) = e^{\frac{x^2+9}{x^2-9}}$$

1

$$D(f) = \{ x \in \mathbb{R} \mid x^2 - 9 \neq 0 \}$$

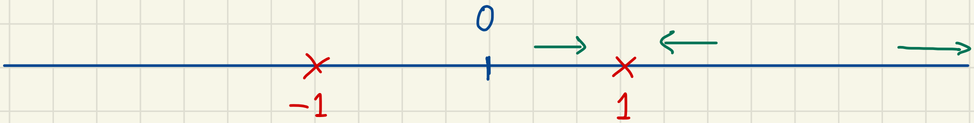
$$= \mathbb{R} \setminus \{ \pm 3 \}$$

2

$$f(-x) = e^{\frac{(-x)^2+9}{(-x)^2-9}} = e^{\frac{x^2+9}{x^2-9}} = f(x)$$

$\Longrightarrow$ f è pari $\Rightarrow$ il grafico di f è simmetrico rispetto l'asse delle y.

Possiamo studiare f solo per $x \geq 0$

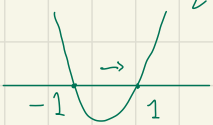


$$f(0) = e^{-1}$$

$$\lim_{x \to 3^-} e^{\frac{x^2+9}{x^2-9}} = 0^+$$

$$\left(\lim_{x \to 3^-} \frac{\overbrace{x^2+9}^{2}}{\underbrace{x^2-9}_{0^-}} = -\infty \right)$$

3^+ $\quad$ $+\infty$



$$\lim_{x \to 3^+} e^{\frac{x^2+9}{x^2-9}} = +\infty$$

$$\lim_{x \to +\infty} e^{\frac{x^2+9}{x^2-9}} = \lim_{x \to +\infty} e^{\frac{1+\frac{9}{x^2}}{1-\frac{9}{x^2}}} = e$$

④

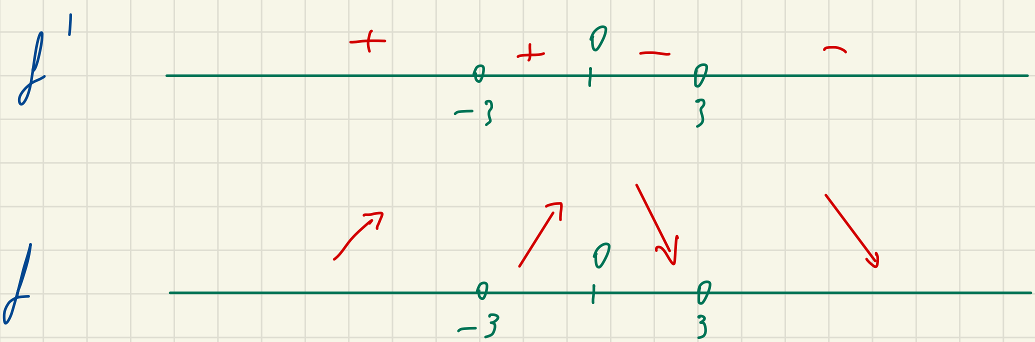
$$f(x) = e^{\frac{x^2+9}{x^2-9}}$$

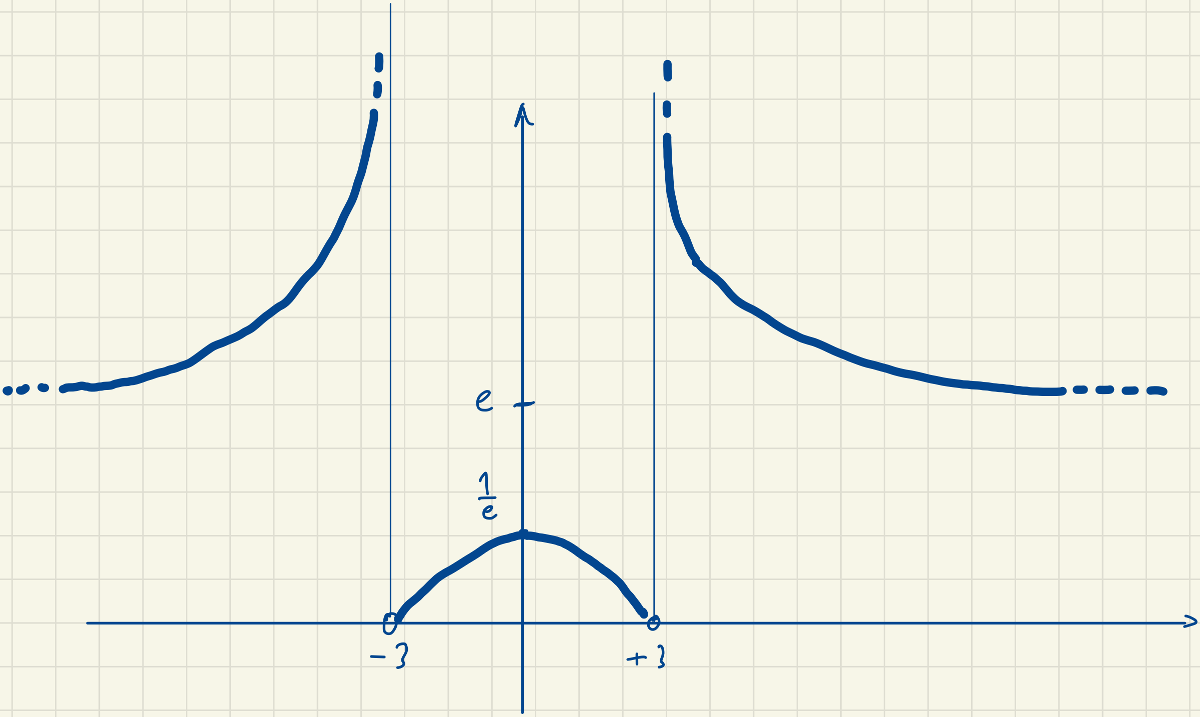
$$f'(x) = e^{\frac{x^2+9}{x^2-9}} \cdot \frac{2x(x^2-9) - 2x(x^2+9)}{(x^2-9)^2} =$$

$$= e^{\frac{x^2+9}{x^2-9}} \cdot \frac{2x^3 - 18x - 2x^3 - 18x}{(x^2-9)^2} =$$

$$= \frac{e^{\frac{x^2+9}{x^2-9}}}{(x^2-9)^2} \cdot (-36x) \leq 0 \quad \text{si} \quad x \geq 0$$

$$< 0 \quad \text{si} \quad x > 0$$





$$\operatorname{Im} f = \left]0, \frac{1}{e}\right] \cup \left]e, +\infty\right[$$

$\# S = 2$ se $\alpha \in \left]0, \frac{1}{e}\right[\cup \left]e, +\infty\right[$

②

$$\lim_{x\to 0} \frac{\sin(\sin x) - \sin x + \frac{x^3}{6}}{x^5}$$

$\sin x \approx x$

lo teniamo così!

$$\sin(\sin x) = \sin x - \frac{\sin^3 x}{6} + \frac{\sin^5 x}{5!} + o(x^5)$$

lo teniamo così !

$$\sin(\sin x) = \sin x - \frac{\sin^3 x}{6} + \frac{\sin^5 x}{5!} + o(x^5)$$

$$= \sin x - \frac{1}{6}\left(x - \frac{x^3}{6}\right)^3 + \frac{x^5}{5!} + o(x^5) =$$

$$= \sin x - \frac{1}{6}\left(x^3 + 3 \cdot x^2 \cdot \left(-\frac{x^3}{6}\right)\right) + \frac{x^5}{60} + o(x^5)$$

$$= \sin x - \frac{x^3}{6} + \frac{x^5}{12} + \frac{x^5}{120} + o(x^5)$$

$$= \sin x - \frac{x^3}{6} + \frac{11}{120} x^5 + o(x^5)$$

$$\Longrightarrow$$

$$\sin(\sin x) - \sin x = -\frac{x^3}{6} + \frac{11}{120} x^5 + o(x^5)$$

$$\lim_{x \to 0} \frac{\sin(\sin x) - \sin x + \frac{x^3}{6}}{x^5} =$$

$$= \lim_{x \to 0} \frac{\cancel{-\frac{x^3}{6}} + \frac{11}{120} x^5 + o(x^5) + \cancel{\frac{x^3}{6}}}{x^5} =$$

$$= \lim_{x \to 0} \left(\frac{11}{120} + \frac{o(x^5)}{x^5} \right) = \frac{11}{120}$$