15 Giugno 2021

①

$$f(x) = \ln(9x - x^3)$$

$$D(f) = \{ x \mid 9x - x^3 > 0 \}$$

$$9x - x^3 = x(9 - x^2)$$

		-3		0		$+3$	
$9 - x^2$	$-$		$+$		$+$		$-$
x	$-$		$-$	0	$+$		$+$
$9x - x^3$	$+$		$-$		$+$		$-$
		-3		0		3	

$$D(f) = \{ x \in \mathbb{R} \mid x < -3 \ \vee \ 0 < x < 3 \}$$

$$\lim_{x \to -\infty} \ln(\overbrace{9x - x^3}^{+\infty}) = +\infty$$

$$\lim_{x \to -3^-} \ln\left(\overset{0^+}{9x - x^3}\right) = -\infty$$

0^+

3^-

$$f' = \frac{1}{9x - x^3} \cdot (9 - 3x^2)$$

$$f' = 0 \iff 9 - 3x^2 = 0 \iff x = \cancel{\pm}\sqrt{3}$$

ma $-\sqrt{3} \notin D(f)$

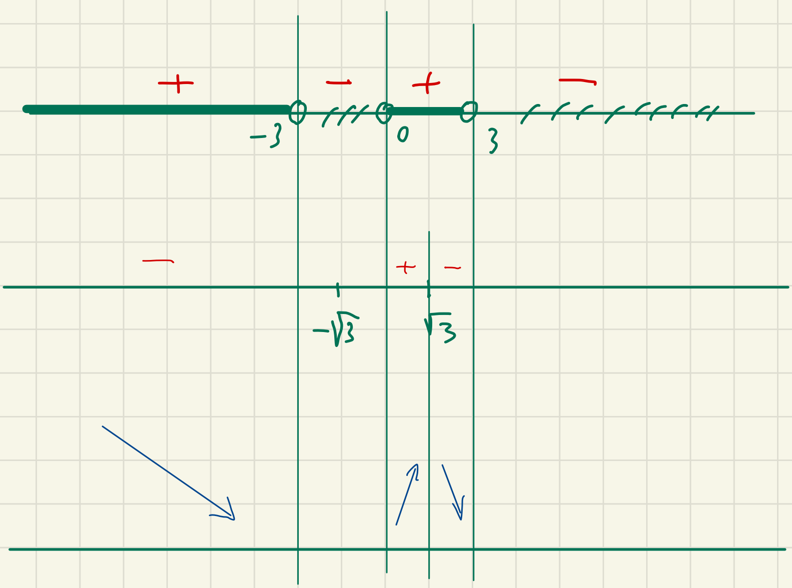
$D(f)$:

$f' > 0$

$\Updownarrow$

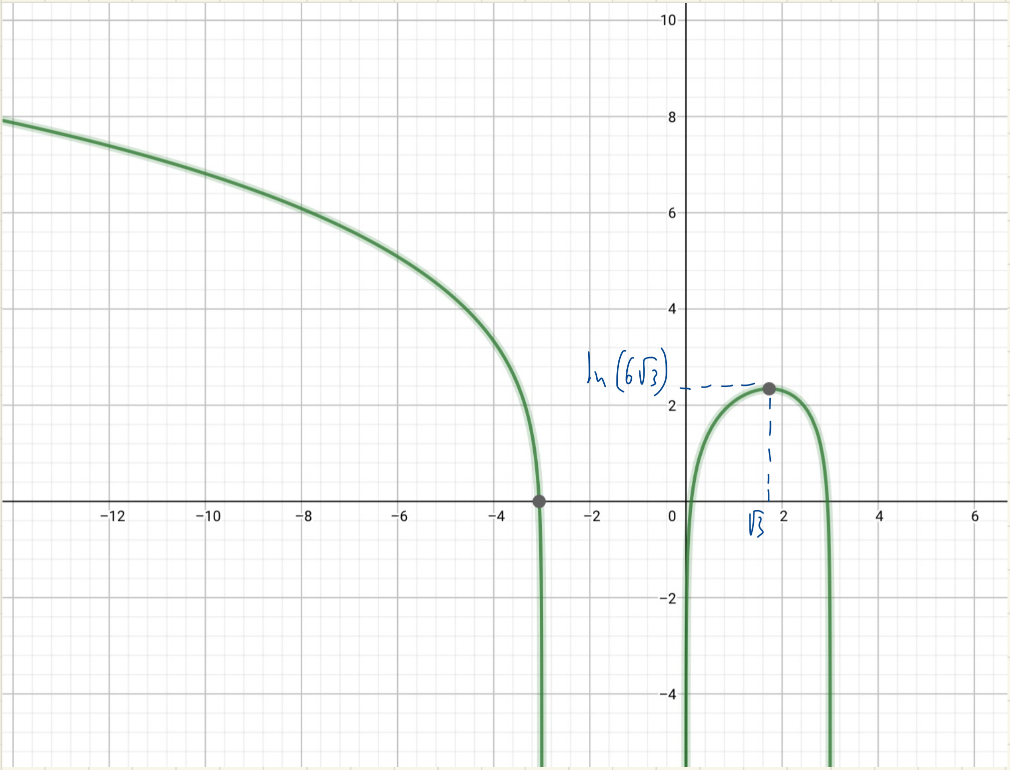
$3 - x^2 > 0$

f



$$f(\sqrt{3}) = \ln\left(9\sqrt{3} - 3\sqrt{3}\right) = \ln 6\sqrt{3}$$

$x = \sqrt{3}$ p. di max relativo



$\operatorname{Im} f = \mathbb{R}$

$f(x) = \lambda$ tre radici $\iff$ $\lambda < \ln(6\sqrt{3})$

(2) $$\lim_{x\to 0} \frac{e^{x\cos x} - e^{\sin x} + \frac{x^3}{3}}{x^4}$$

$$x\cos x = x\left(1 - \frac{x^2}{2} + o(x^3)\right) =$$

$$= x - \frac{x^3}{2} + o(x^4)$$

$$e^{x\cos x} = 1 + \left(x - \frac{x^3}{2} + o(x^4)\right) +$$

$$+ \frac{1}{2!}\left(x - \frac{x^3}{2} + o(x^4)\right)^2 + \frac{1}{3!}\left(x + o(x^2)\right)^3$$

$$+ \frac{1}{4!}\left(x + o(x)\right)^4 =$$

$$= 1 + x - \frac{x^3}{2} + \frac{1}{2}\left(x^2 - x^4\right) +$$

$$+ \frac{x^3}{6} + \frac{x^4}{24} + o(x^4)$$

$$= 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{11}{24}x^4 + o(x^4)$$

$$e^{\sin x} = 1 + \sin x + \frac{\sin^2 x}{2} + \frac{\sin^3 x}{3!} + $$

$$+ \frac{\sin^4 x}{4!} + o(x^4)$$

$$= 1 + \left(x - \frac{x^3}{3!} + o(x^4)\right) + \frac{\left(x - \frac{x^3}{6} + o(x^3)\right)^2}{2} +$$

$$+ \frac{\left(x + o(x^2)\right)^3}{6} + \frac{\left(x + o(x)\right)^4}{24} + o(x^4) =$$

$$= 1 + x - \frac{x^3}{6} + \frac{x^2}{2} - \frac{x^4}{6} + \frac{x^3}{6} +$$

$$+ \frac{x^4}{24} + o(x^4)$$

$$= 1 + x + \frac{x^2}{2} - \frac{x^4}{8} + o(x^4)$$

$$\frac{e^{x\cos x} - e^{\sin x} + \frac{x^3}{3}}{x^4} =$$

$$= \frac{\cancel{1} + \cancel{x} + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^3}{3}} - \frac{11}{24}x^4 - \cancel{1} - \cancel{x} - \cancel{\frac{x^2}{2}} + \frac{x^4}{8} + \cancel{\frac{x^3}{3}} + o(x^4)}{x^4}$$

$$= \frac{\left(-\frac{11}{24} + \frac{1}{8}\right)x^4 + o(x^4)}{x^4} =$$

$$= -\frac{1}{3} + \frac{o(x^4)}{x^4} \xrightarrow[x \to 0]{} -\frac{1}{3}$$